

# Internal Friday Seminar

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- Personal info
- Teaching
- Research
  - M-numbers
  - Distributed Control
  - Underactuated Systems

## Sweden

Lund University, LTH, Automatic control, 01\10\2021.

- Arbetsförmedlingen → Academicum company (*Korta vägen* education) → **Practice** and **Swedish language**.
- Supervisors: Anders Robertsson and Charlotta Johnsson.

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## Croatia

Zagreb University, FSB<sup>a</sup>, Automatic control <sup>b</sup>, 2004.-2019.

- 2003. MSc mechanical engineer, energy direction
- 2010. PhD automation (field: control system theory)
- 2004–2019. Courses: electrical engineering, programming

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<sup>a</sup>Faculty of mechanical engineering and naval architecture, Zagreb, Croatia

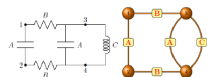
<sup>b</sup>@ Department of Robotics and Production System Automation



# M-system theory and M-numbers

<http://m-system.fsb.hr>

## M-system theory<sup>1</sup>.



◊  $Z_{12} = A | PS | (B | SS | ((C | PS | A) | SS | B)) = BCABA$   
◊  $Z_{13} = B | PS | (A | SS | B | SS | (A | PS | C)) = BACAB$   
◊  $Z_{14} = (A | SS | B) | PS | (B | SS | (A | PS | C)) = BACAB$   
◊  $Z_{23} = Z_{14}$ ,  $Z_{24} = Z_{13}$   
◊  $Z_{34} = C | PS | A | PS | (B | SS | A | SS | B) = CABA$

### M-system

**Alphabet  $\Gamma$ :**  $\alpha < \dots < a_1 < \dots < a_2 < \dots < \omega$

**M-word:** pairs of (concatenated, ) symbols from  $\Gamma$   
i.e.  $(a_1, a_2) \rightarrow a_1 \cdot a_2 \rightarrow a_1 a_2$

**M-logical values:** the most left symbol  $l(x)$   
+ the most right symbol  $r(x)$  of m-word  $x$ ,  
i.e.  $(l, r) = q(x)$

**Dual operator:**  $D(a_1) = a_2$ , iff  
 $\delta(a_1, \alpha) = \delta(a_2, \omega) \rightarrow \delta(a_1, \omega) = \delta(a_2, \alpha)$

**Negation operator:** the unary, complement operator  
 $K(x) =_{def} l'(x) \cdot x \cdot r'(x) = l' \cdot x \cdot r'$   
where  $l'(x) = D(l(x))$  and  $r'(x) = D(r(x))$

**Infimum and supremum:**  $a \downarrow b = a$ ,  $a \uparrow b = b$ ,  
where  $a < b$

**M-logic** = negation ( $K$ ) + conjunction ( $\wedge$ )

operator	symbol	rule
negation	$\neg$	$(l'_x, r'_x)$
conjunction	$\wedge$	$(l_x \downarrow l_y, r_x \uparrow r_y)$
disjunction	$\vee$	$(l_x \uparrow l_y, r_x \downarrow r_y)$
p-implication	$\xrightarrow{p}$	$(l'_x \uparrow l_y, r'_x \downarrow r_y)$
s-implication	$\xrightarrow{s}$	$(l'_x \downarrow l_y, r'_x \uparrow r_y)$

## Further theory development :

- Semantic of natural language<sup>2</sup>
- M-valued logic
- Topology of electric circuits
- Dynamical systems and optimization?

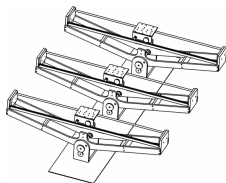
<sup>1</sup>Invented by Miroslav Šare: *Jorbologija*, Element Zagreb, 2000.

<sup>2</sup>Mario Essert, Ivana Kuzmanović, Ivan Vazler, Tihomir Žilić: *Theory of M-systems*, Logic journal of the IGPL, 25(5):836-858, August 2017.

# Distributed control

Platform for education, testing and verification of real-life implementation of decentralized and distributed control solutions.

Three spring connected seesaw-cart subsystems<sup>3</sup>.



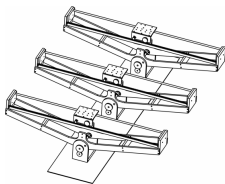
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<sup>3</sup>M. Lobrović, A. Jokić, V. Milić, T. Žilić, M. Jokić, J. Kasać, Z. Domitran, M. Crneković: *A Case Study in Distributed Control: Elastically Interconnected Seesaw-cart Systems.*, 25th Mediterranean Conference on Control and Automation, 2017.

# Distributed control

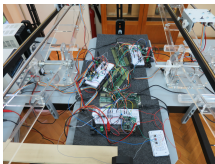
Platform for education, testing and verification of real-life implementation of decentralized and distributed control solutions.

Three spring connected seesaw-cart subsystems<sup>3</sup>.



My participation:

- electronics
- programming
- micro-controllers



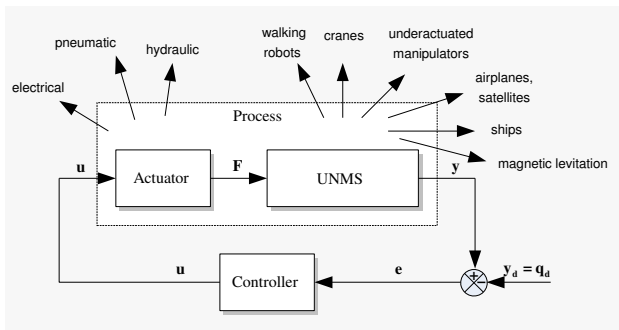
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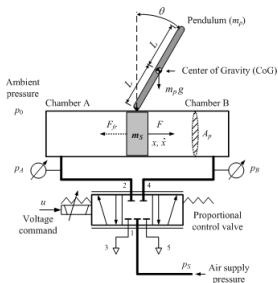
# PhD research - underactuated systems

- Underactuated nonlinear mechanical systems (UNMS)
  - actuators < degrees of freedom (DOF)
  - mechanical systems with input coupling
  - fully actuated but with actuators' failure → UNMS
- Included actuators dynamics. Actuators of higher orders, nonlinearities, discontinuous friction.

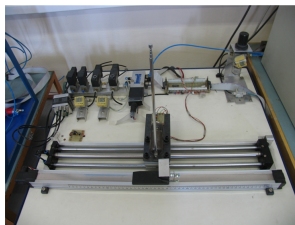


# PhD research - motivation

## Pneumatically actuated slider-inverted pendulum



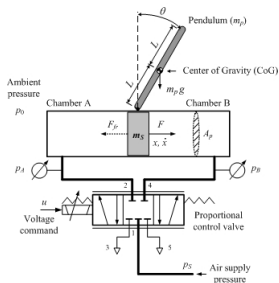
(a) Scheme



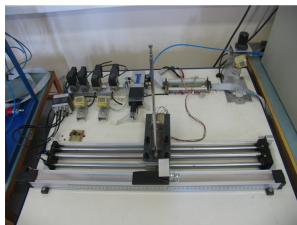
(b) Laboratory setup

# PhD research - motivation

## Pneumatically actuated slider-inverted pendulum



(c) Scheme



(d) Laboratory setup

### Control challenge

Simultaneous stabilization and trajectory of arbitrarily selected degrees of freedom.<sup>a</sup>

<sup>a</sup>**Simultaneous stabilization and trajectory tracking** refers to a closed control loop where some DOF follow desired trajectories while simultaneously other DOF are stabilized.

# Euler-Lagrange equations of UNMS

$$\begin{aligned}\frac{d}{dt} \frac{\partial \mathcal{L}(\mathbf{q}, \dot{\mathbf{q}})}{\partial \dot{\mathbf{q}}_a} - \frac{\partial \mathcal{L}(\mathbf{q}, \dot{\mathbf{q}})}{\partial \mathbf{q}_a} + \mathbf{F}_{dis_a}(\mathbf{q}, \dot{\mathbf{q}}) &= \mathbf{H}_a(\mathbf{q})\mathbf{F} \\ \frac{d}{dt} \frac{\partial \mathcal{L}(\mathbf{q}, \dot{\mathbf{q}})}{\partial \dot{\mathbf{q}}_u} - \frac{\partial \mathcal{L}(\mathbf{q}, \dot{\mathbf{q}})}{\partial \mathbf{q}_u} + \mathbf{F}_{dis_u}(\mathbf{q}, \dot{\mathbf{q}}) &= \mathbf{H}_u(\mathbf{q})\mathbf{F}\end{aligned}$$

$\mathbf{q} = [\mathbf{q}_a, \mathbf{q}_u]^T$ ,  $\mathbf{q} \in \mathbb{R}^n$ ,  $\mathbf{q}_a \in \mathbb{R}^{n_a}$ ,  $\mathbf{q}_u \in \mathbb{R}^{n_u}$ ,  $\mathbf{F} \in \mathbb{R}^{n_a}$

Subscripts:  $a$  - **actuated**,  $u$  - **unactuated** DoF

**Underactuated systems:  $n_a < n$**

2. order nonholonomic systems:  $\mathbf{H}_a(\mathbf{q}) = \mathbf{I}$ ,  $\mathbf{H}_u(\mathbf{q}) = \mathbf{0}$   
UNMS with input coupling:  $\det(\mathbf{H}_a(\mathbf{q})) \neq 0$ ,  $\mathbf{H}_u(\mathbf{q}) \neq \mathbf{0}$ .

# State space of coupled system (UNMS+actuators)

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$
$$\mathbf{y} = \gamma(\mathbf{x})$$

$$\underbrace{\begin{bmatrix} \dot{\mathbf{x}}_{qa1} \\ \dot{\mathbf{x}}_{qu1} \\ \dot{\mathbf{x}}_{qa2} \\ \dot{\mathbf{x}}_{qu2} \\ \dot{\mathbf{x}}_{ac} \end{bmatrix}}_{\mathbf{\dot{x}}} = \underbrace{\begin{bmatrix} \mathbf{x}_{qa2} \\ \mathbf{x}_{qu2} \\ \mathbf{f}_3(\mathbf{x}_q) + \mathbf{f}_{dis3}(\mathbf{x}_q) + \mathbf{B}_3(\mathbf{x}_q)\gamma_{ac}(\mathbf{x}_{ac}, \mathbf{x}_q) \\ \mathbf{f}_4(\mathbf{x}_q) + \mathbf{f}_{dis4}(\mathbf{x}_q) + \mathbf{B}_4(\mathbf{x}_q)\gamma_{ac}(\mathbf{x}_{ac}, \mathbf{x}_q) \\ \mathbf{f}_{ac}(\mathbf{x}_q, \mathbf{x}_{ac}) \end{bmatrix}}_{\mathbf{f}(\mathbf{x})} + \underbrace{\begin{bmatrix} \mathbf{0}_{n_a \times n_a} \\ \mathbf{0}_{n_u \times n_a} \\ \mathbf{0}_{n_a \times n_a} \\ \mathbf{0}_{n_u \times n_a} \\ \mathbf{g}_{ac}(\mathbf{x}_{ac}) \end{bmatrix}}_{\mathbf{g}(\mathbf{x})} \underbrace{\mathbf{u}}_{\mathbf{u}_{ac}}$$
$$\mathbf{y} = \gamma(\mathbf{x})$$

# Controller

Based on sliding surface control methodology

Sliding surface  $\mathbf{s}$ :

$$\begin{aligned}\mathbf{s} &= \dot{\mathbf{e}} + \lambda_1 \mathbf{e} = \mathbf{0}, \text{ UNMS+actuator}(r=0) \\ \mathbf{s} &= \ddot{\mathbf{e}} + \lambda_2 \dot{\mathbf{e}} + \lambda_1 \mathbf{e} = \mathbf{0}, \text{ UNMS+actuator}(r=1) \\ \mathbf{s} &= \dddot{\mathbf{e}} + \lambda_3 \ddot{\mathbf{e}} + \lambda_2 \dot{\mathbf{e}} + \lambda_1 \mathbf{e} = \mathbf{0}, \text{ UNMS+actuator}(r=2) \\ &\dots\end{aligned}$$

First equation: If  $\lambda_1 \succ \mathbf{0}$ , then  $\mathbf{e} \rightarrow \mathbf{0}$  when  $t \rightarrow \infty$ .

Mark  $r$  - actuator's relative degree regarding to  $\mathbf{y}_{ac}$ .

$$\mathbf{y} = \begin{bmatrix} \mathbf{x}_{qa1} \\ \mathbf{x}_{qu1} \end{bmatrix}, \quad \mathbf{y}_d = \begin{bmatrix} (\mathbf{x}_{qa1})_d \\ (\mathbf{x}_{qu1})_d \end{bmatrix}, \quad \mathbf{e} = \mathbf{y} - \mathbf{y}_d = \begin{bmatrix} \mathbf{e}_a \\ \mathbf{e}_u \end{bmatrix}$$

$$\ddot{\mathbf{e}} = \begin{bmatrix} \mathbf{f}_3(\mathbf{x}_q) + \mathbf{f}_{dis_3}(\mathbf{x}_q) + \mathbf{B}_3(\mathbf{x}_q)\gamma_{ac}(\mathbf{x}_{ac}, \mathbf{x}_q) - (\ddot{\mathbf{x}}_{qa1})_d \\ \mathbf{f}_4(\mathbf{x}_q) + \mathbf{f}_{dis_4}(\mathbf{x}_q) + \mathbf{B}_4(\mathbf{x}_q)\gamma_{ac}(\mathbf{x}_{ac}, \mathbf{x}_q) - (\ddot{\mathbf{x}}_{qu1})_d \end{bmatrix}$$

# Controller

Example for UNMS+actuator( $r=1$ )

Equivalent control laws:  $(\mathbf{u}_{eq})_a$  - actuated,  $(\mathbf{u}_{eq})_u$  - unactuated

$$\begin{bmatrix} (\mathbf{u}_{eq})_a \\ (\mathbf{u}_{eq})_u \end{bmatrix} = \begin{bmatrix} - \left[ \mathbf{B}_3(\mathbf{x}_q) \frac{\partial \gamma_{ac}(\mathbf{x}_{ac}, \mathbf{x}_q)}{\partial \mathbf{x}_{ac}} \mathbf{g}_{ac}(\mathbf{x}_{ac}) \right]^{-1} \left\{ \mathbf{Z}_3 + \mathbf{B}_3(\mathbf{x}_q) \frac{\partial \gamma_{ac}(\mathbf{x}_{ac}, \mathbf{x}_q)}{\partial \mathbf{x}} \mathbf{f}(\mathbf{x}) + \chi_a(\mathbf{s}_a) \right\} \\ - \left[ \mathbf{B}_4(\mathbf{x}_q) \frac{\partial \gamma_{ac}(\mathbf{x}_{ac}, \mathbf{x}_q)}{\partial \mathbf{x}_{ac}} \mathbf{g}_{ac}(\mathbf{x}_{ac}) \right]^+ \left\{ \mathbf{Z}_4 + \mathbf{B}_4(\mathbf{x}_q) \frac{\partial \gamma_{ac}(\mathbf{x}_{ac}, \mathbf{x}_q)}{\partial \mathbf{x}} \mathbf{f}(\mathbf{x}) + \chi_u(\mathbf{s}_u) \right\} \end{bmatrix}$$

$\chi(\mathbf{s}) \in \{ \alpha \text{sign}(\mathbf{s}), \alpha \tanh(\mathbf{s}), \alpha \text{sat}(\mathbf{s}), \dots \}, \alpha > \mathbf{0}$

Proposed control law for combined control:

$$\mathbf{u} = \varphi_a \cdot (\mathbf{u}_{eq})_a + \varphi_u \cdot (\mathbf{u}_{eq})_u$$

$\varphi_a, \varphi_u \in \mathbb{R}^{n_a \times n_a}$  are diagonal weight matrices.

- [1] T. Zilic, J. Kasac, Z. Situm, and M. Essert. *Simultaneous stabilization and trajectory tracking of underactuated mechanical systems with included actuators dynamics*. *Multibody System Dynamics*, vol. 29, no. 1, pp. 1–19, 2013.
- [2] T. Zilic, J. Kasac, Z. Situm, and M. Essert. *Application of SSTT control algorithm for underactuated: surface vessel, PPR manipulator and pneumatically actuated slider-inverted pendulum*. *Advanced Robotics*, vol. 28, no. 8, pp. 545–556, 2014.
- [3] T. Zilic, D. Pavkovic, and D. Zorc. *Modeling and control of a pneumatically actuated inverted pendulum*. *ISA Transactions*, vol. 48, no. 3, pp. 327–335, 2009.



Thank you! Tack så mycket! Hvala vam!

Thank you! Tack så mycket! Hvala vam!

inspCoffe

Time for questions!