Control of an exponential disturbance

a semi-simple problem

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Outline

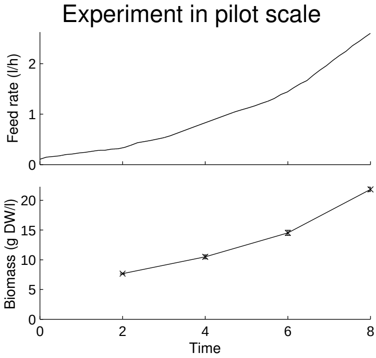
The problem

A solution

Proving the solution

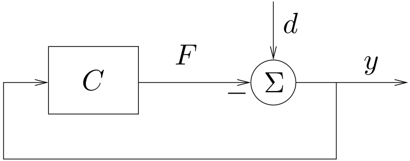
The problem - motivating background

- Fed-batch process.
- Exponential growth is expected and desired.
- Tracking to keep up with feed demand.

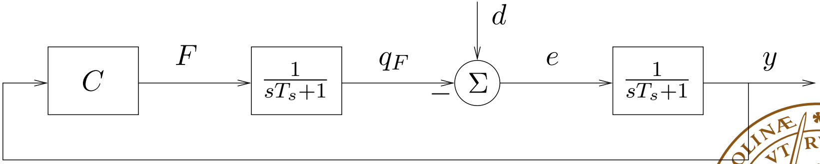


The problem - modelled

Simplest possible setup:



Setup including dynamics:



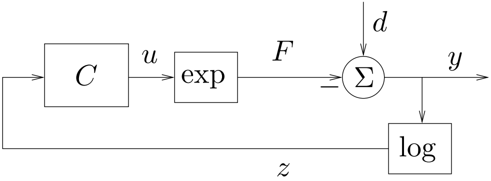
It is an extremely simple problem!

Solving the problem

We want a simple solution to this simple problem!

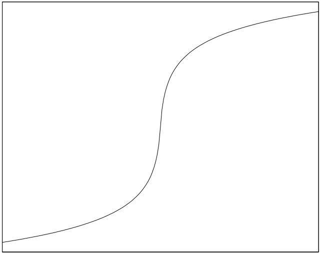
- The disturbance is the exponential of a ramp.
- Can perhaps be "counteracted" by logarithmizing the measurement?
  $z = \mathrm{sign}(y)\log(|y| + 1)$
- Maybe we can go full circle by using an exponential on the control signal?
  $F = \mathrm{sign}(u)(\exp(|u|) - 1)$

Modified set-up



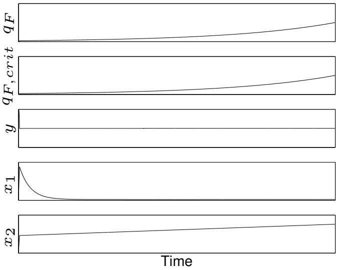
Exp function

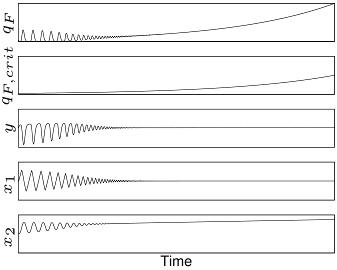
Log function



Can a simple controller achieve stability?

Simulated solutions





Simplest possible setup:
$u = K_c \frac{sT_c+1}{s^2T_c}$

Setup including dynamics:
$u = K_c(\frac{sT_c+1}{sT_c})^2$
for $T_c > T_s$

Case closed? (Hint: no)

Proof of stability

- Nice simple solution to a simple problem.
- We want a proof of stability.
- Maybe the proof is just as simple? (Hint: no again)

What is so difficult?

We start by keeping it simple:

- Setup without process dynamics.
- Only one controller zero ($u = K_c \frac{sT_c+1}{s^2 T_c}$).
- Only positive values considered, no need for sign and abs functions.
- $F = exp(u)$ rather than $F = exp(u) - 1$.

$$
\begin{aligned}
y &= \exp(\mu t) - F \\
\dot{x}_1 &= \log(y+1) \\
\dot{x}_2 &= x_1 \\
F &= \exp(K_c(T_c x_1 + x_2))
\end{aligned}
$$

Convergence to a solution?

The system becomes

$$\dot{x}_1 = \log(\exp(\mu t) - \exp(K_c(T_c x_1 + x_2) + 1)$$
$$\dot{x}_2 = x_1$$

For $y = 0$, we get $\mu t = K_c(T_c x_1 + x_2)$ (and $\dot{x}_1 = 0$).
And hence

$$x_1 = \mu / K_c$$
$$x_2 = \mu / K_c(t - T_c)$$

Lyapunov function to show convergence to this trajectory?
No luck so far.

How to prove it?

- Simple methods? Would of course be ideal, maybe I have missed something?
- Less simple methods? (Contraction theory suggested by Anders Rantzer)
- Either way, a proof of stability would almost certainly be publishable.
- Perhaps the audience has some ideas?