Ball and Finger System

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Friday Seminar
March 18th, 2016

Outline

- Spring
- Introduction
- Modeling
- Optimal Trajectory
- Conclusion and Future Research

Spring

- Astronomical Spring
- Northward Equinox (vernal in the northern hemisphere)
- Ancient Spring Festival (Nouruz)

Damavand peak, Iran

Estahban, Iran

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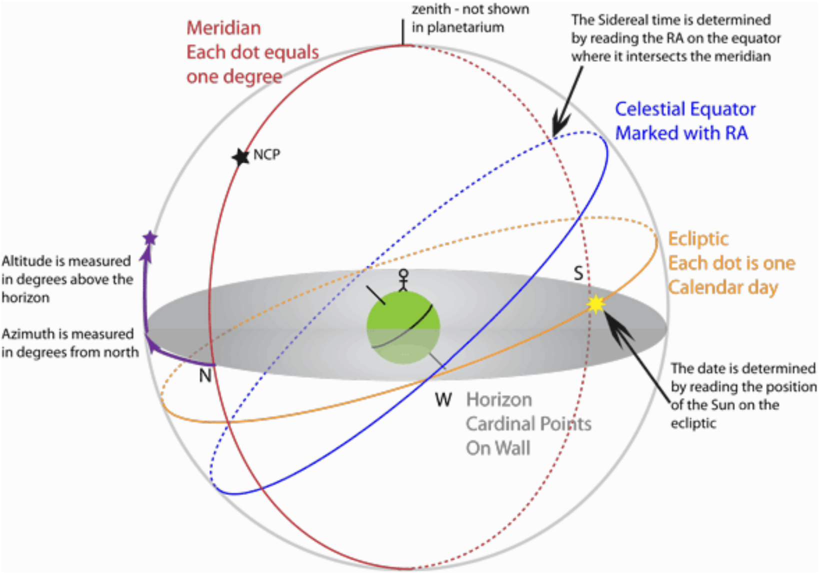
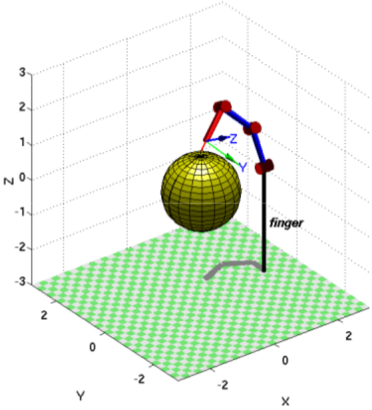
20th March 2016, 05:30:12 CET!



Introduction



Motivation

- Understanding the physical limitations of the problem
- Enhancing existing models
  - Analytic 3D models with contacts
  - Rolling of finger against ball
  - Dynamics
  - Slippage
  - Suitable for simulation and optimization
- Optimal trajectories
  - Integrated paradigm
  - Tuning of the model rather than the cost function

Introduction

State of the Art

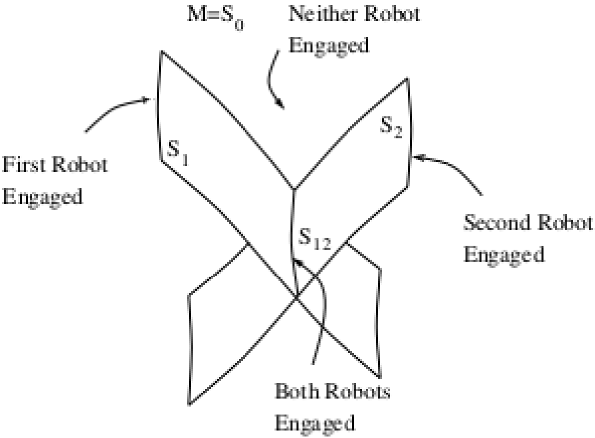
- Kinematic models
- Hand-tuned controllers and Heuristics
- Reinforcement learning
- Advanced physics simulator and i-LQG

Introduction

Notions:

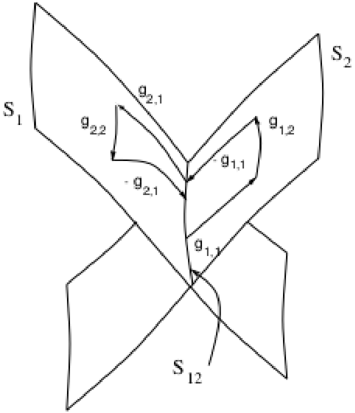
- Stratified System
- Non-smooth Dynamics
  - Dynamics with Contacts
  - Unilateral Constraint
- Non-holonomic Constraint

Stratified System



Configuration manifold structure of two cooperating robots

Stratified System



Sequence of flows

Non-smooth Dynamics

$$\dot{x} = g(x, u) \tag{1}$$

$$f(x, t) \geq 0 \tag{2}$$

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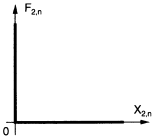
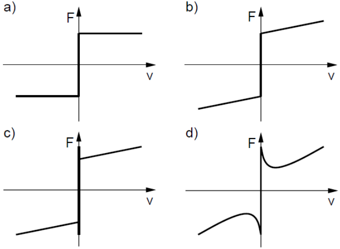
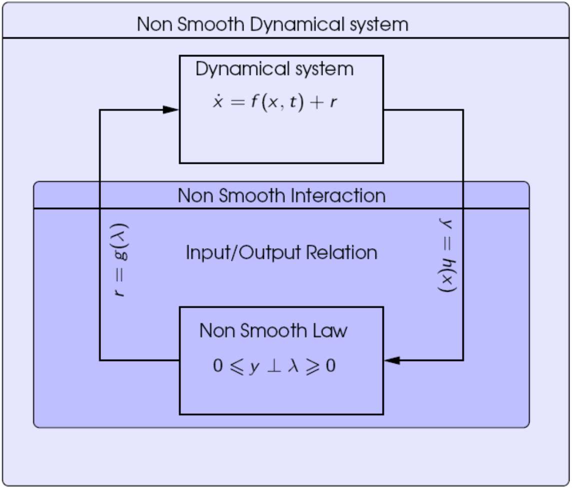


Corner Law



Friction Models

Modeling Principle



Non-holonomic Constraint

$$\sum_{k=1}^{n} a_k^j(q)\dot{q}^k = 0$$

where $j = 1, \cdots, m$.

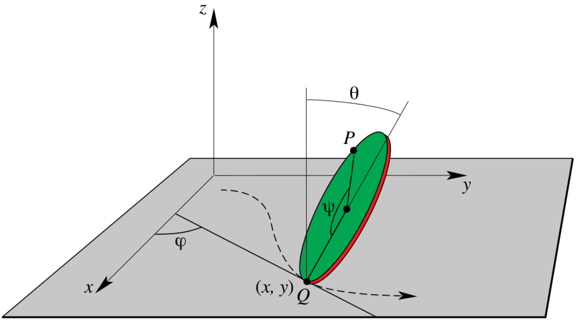
Holonomic if there is

$$b^j(q) = 0$$

such than

$$\sum_{k=1}^{n} \frac{\partial b^j}{\partial q^k}\dot{q}^k = 0.$$

Non-holonomic Constraint



Difficulties

- Stiff or Discontinuous
- Painlevé paradox
  - Indeterminacy
  - Inconsistency

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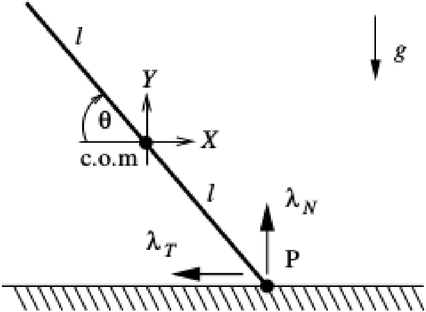


Ingredients of the Model

- Forward and Inverse Kinematics
- Collision Detection
- Constraints
- Dynamics
- Transitions

Let’s watch

- Finger falling
- Finger slipping on the ball
- Finger slipping and rolling on the ball
- Finger sticking to the ball
- Transitions

Dynamics

Differential Algebraic Equations (DAE) of Index-2

$$M_f(q)\ddot{q} + C(q,\dot{q})\dot{q} + \mu_v\dot{q} + g(q) = \tau + J_c^T\lambda$$
$$I_b\dot{\omega} + \omega \times I_b\omega + D\omega = -r_c \times \lambda.$$

- Free motion:

$$\lambda = 0 \quad (3)$$

- Sticking:

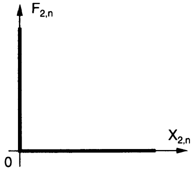
$$J_c\dot{q} + r_c \times \omega = 0 \quad (4)$$

- Slipping:

$$r_c \cdot J_c\dot{q} = 0 \quad (5)$$
$$\lambda \cdot \Delta v = \|\Delta v\| \, \mu_k(\lambda \cdot n) \quad (6)$$
$$(n \times \Delta v) \cdot \lambda = 0 \quad (7)$$

Translation to Hybrid System

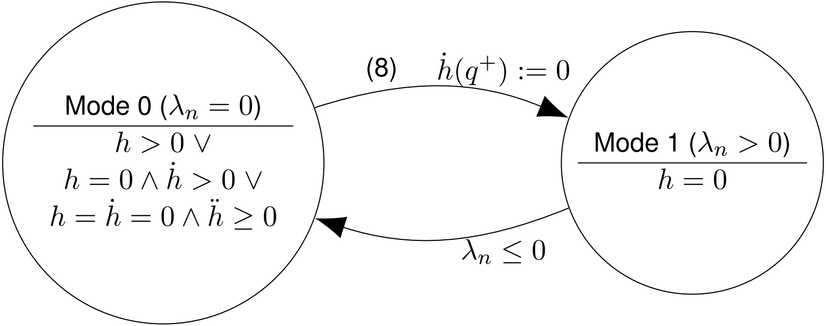
$$\begin{cases} h \cdot \lambda_n = 0 \\ h \geq 0,\ \lambda_n \geq 0. \end{cases}$$



where

$$h(q) := \|r_c\| - r \leq 0$$

Translation to Hybrid System



$$h < 0 \;\vee (h = 0 \wedge \dot{h} < 0) \;\vee (h = \dot{h} = 0 \wedge \ddot{h} < 0) \qquad (8)$$

Optimal Trajectories

$$J = x^T S x|_{t_f} + \int_{t_0}^{t_f} (x - x_0)^T Q (x - x_0) + Q_2 x + u^T R u \, dt$$

- Implementation
  - C++
  - PSOPT (with IPOPT)
  - Direct collocation methods
- Handling switching
  - Multiphase
  - smooothing

Multiphase Optimization

Find the control trajectories, $u^{(i)}(t), t \in [t_0^{(i)}, t_f^{(i)}]$, state trajectories $x^{(i)}(t), t \in [t_0^{(i)}, t_f^{(i)}]$, static parameters $p^{(i)}$, and times $t_0^{(i)}, t_f^{(i)}$, $i = 1, \ldots, N_p$, to minimize the following performance index:

$$J = \sum_{i=1}^{N_p} \left[\varphi^{(i)}[x^{(i)}(t_f^{(i)}), p^{(i)}, t_f^{(i)}] + \int_{t_0^{(i)}}^{t_f^{(i)}} L^{(i)}[x^{(i)}(t), u^{(i)}(t), p^{(i)}, t] dt \right]$$

subject to the differential constraints:

$$\dot{x}^{(i)}(t) = f^{(i)}[x^{(i)}(t), u^{(i)}(t), p^{(i)}, t],\ t \in [t_0^{(i)}, t_f^{(i)}],$$

the path constraints

$$h_L^{(i)} \leq h^{(i)}[x^{(i)}(t), u^{(i)}(t), p^{(i)}, t] \leq h_U^{(i)},\ t \in [t_0^{(i)}, t_f^{(i)}],$$

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Multiphase Optimization

the event constraints:

$$e_L^{(i)} \leq e^{(i)}[x^{(i)}(t_0^{(i)}), u^{(i)}(t_0^{(i)}), x^{(i)}(t_f^{(i)}), u^{(i)}(t_f^{(i)}), p^{(i)}, t_0^{(i)}, t_f^{(i)}] \leq e_U^{(i)},$$

the phase linkage constraints:

$$\Psi_l \leq \Psi[x^{(1)}(t_0^{(1)}), u^{(1)}(t_0^{(1)}), \quad (9)$$

$$x^{(1)}(t_f^{(1)}), u^{(1)}(t_f^{(1)}), p^{(1)}, t_0^{(1)}, t_f^{(1)}, \quad (10)$$

$$x^{(2)}(t_0^{(2)}), u^{(2)}(t_0^{(2)}) \quad (11)$$

$$,x^{(2)}(t_f^{(2)}), u^{(2)}(t_f^{(2)}), p^{(2)}, t_0^{(2)}, t_f^{(2)}, \quad (12)$$

$$\vdots \quad (13)$$

$$x^{(N_p)}(t_0^{(N_p)}), u^{(N_p)}(t_0^{(N_p)}), \quad (14)$$

$$x^{(N_p)}(t_f^{(N_p)}), u^{(N_p)}(t_f^{(N_p)})), p^{(N_p)}, t_0^{(N_p)}, t_f^{(N_p)}] \leq \Psi_u \quad (15)$$

Multiphase Optimization

the bound constraints:

$$u_L^{(i)} \leq u^i(t) \leq u_U^{(i)},\ t \in [t_0^{(i)}, t_f^{(i)}],$$

$$x_L^{(i)} \leq x^i(t) \leq x_U^{(i)},\ t \in [t_0^{(i)}, t_f^{(i)}],$$

$$p_L^{(i)} \leq p^{(i)} \leq p_U^{(i)},$$

$$\underline{t}_0^{(i)} \leq t_0^{(i)} \leq \bar{t}_0^{(i)},$$

$$\underline{t}_f^{(i)} \leq t_f^{(i)} \leq \bar{t}_f^{(i)},$$

and the following constraints:

$$t_f^{(i)} - t_0^{(i)} \geq 0,$$

Smoothing

A useful approximation

$$|x| \approx \sqrt{x^2 + \epsilon^2}$$

where ϵ is a small number.

$$\min(a, b) = \frac{1}{2}(a + b - |a - b|)$$

$$\max(a, b) = -\min(-a, -b)$$

$$\text{sat}(x, L, U) := \max(\min(x, U), L)$$

$$\Pi(x, L, U) := \frac{d\,\text{sat}(x, U, L)}{dx}$$

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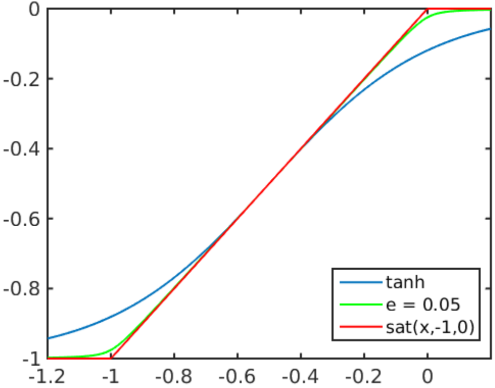
Saturation function



Example Problems

- Reach the ball
- Rotate the ball along an axis as fast as possible
- Show me a point on the ball
- Find a periodic solution to keep the ball rotating

Conclusion and Future Research

- Non-smooth mechanics
- Modeling challenges
- Multiphase optimization

- Handling impacts
- Beyond rigid-body models
- Boot-strapping
- Combining with sample based optimization
- Designing controllers

Thank you for listening!

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