Gravitational Waves and Control

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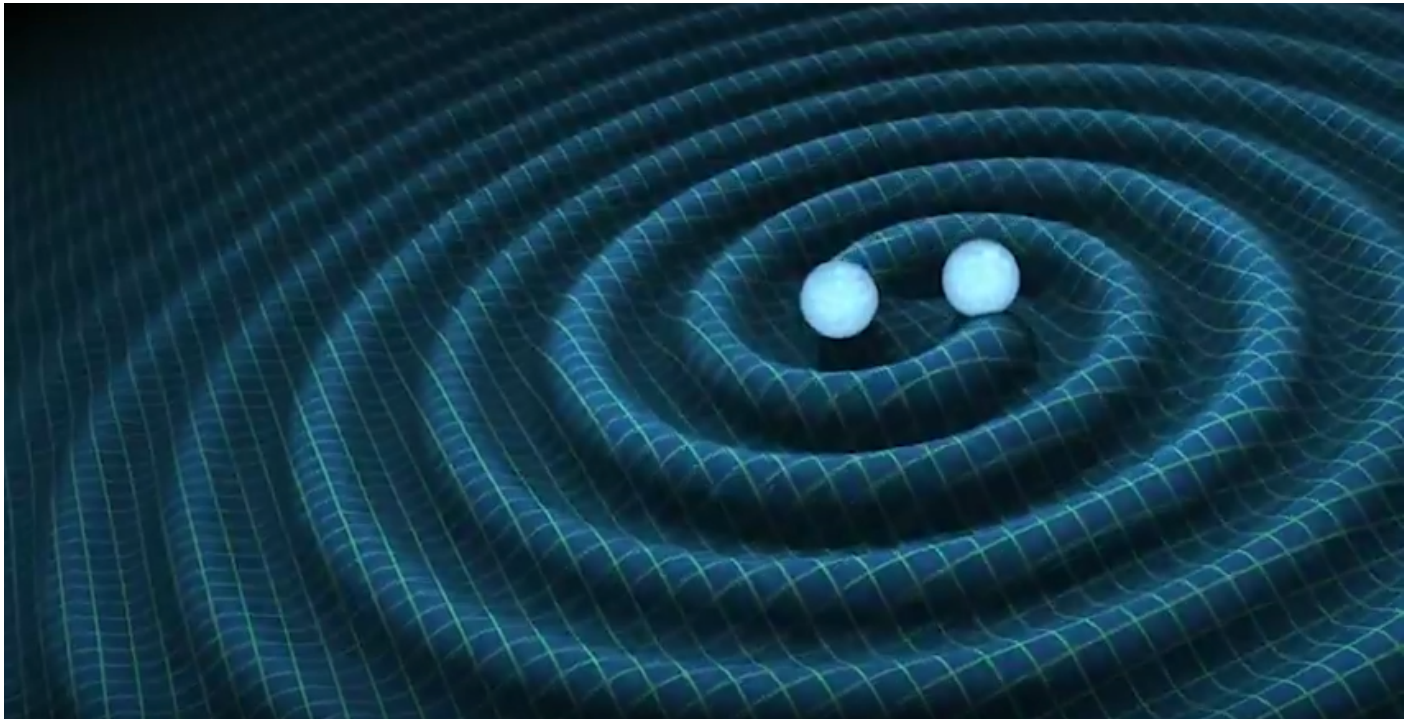
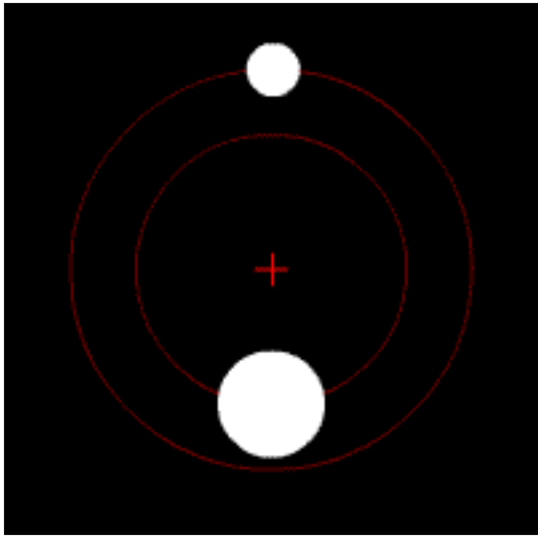
Gravitational Waves



Gravitational Waves



Power radiated by orbiting bodies [edit]

Gravitational waves carry energy away from their sources and, in the case of orbiting bodies, this is associated with an inspiral or decrease in orbit. Imagine for example a simple system of two masses — such as the Earth-Sun system — moving slowly compared to the speed of light in circular orbits. Assume that these two masses orbit each other in a circular orbit in the *x*–*y* plane. To a good approximation, the masses follow simple Keplerian orbits. However, such an orbit represents a changing quadrupole moment. That is, the system will give off gravitational waves.

Suppose that the two masses are m_1 and m_2, and they are separated by a distance r. The power given off (radiated) by this system is:

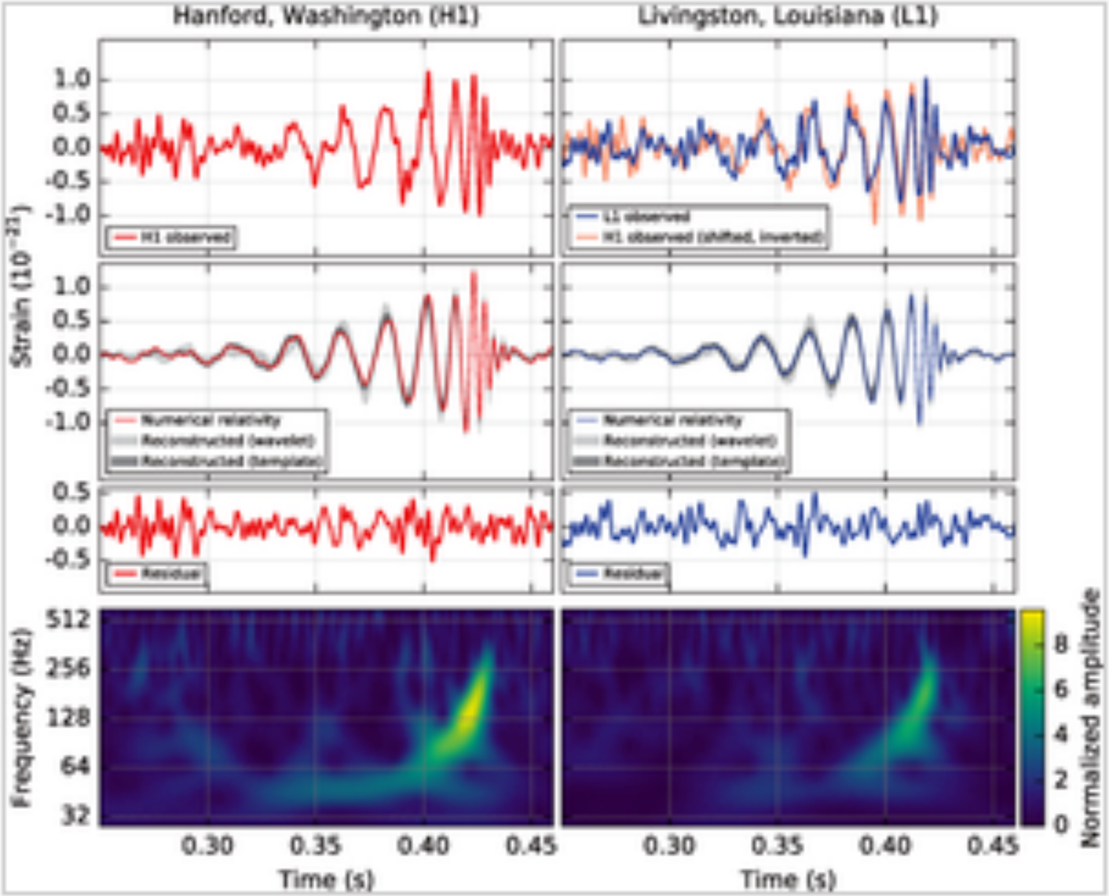
$$P = \frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{32}{5}\frac{G^4}{c^5}\frac{(m_1 m_2)^2(m_1+m_2)}{r^5},$$ [19]

where *G* is the gravitational constant, *c* is the speed of light in vacuum and where the negative sign means that power is being given off by the system, rather than received. For a system like the Sun and Earth, r is about 1.5×10^{11} m and m_1 and m_2 are about 2×10^{30} and 6×10^{24} kg respectively. In this case, the power is about 200 watts. This is truly tiny compared to the total electromagnetic radiation given off by the Sun (roughly 3.86×10^{26} watts).

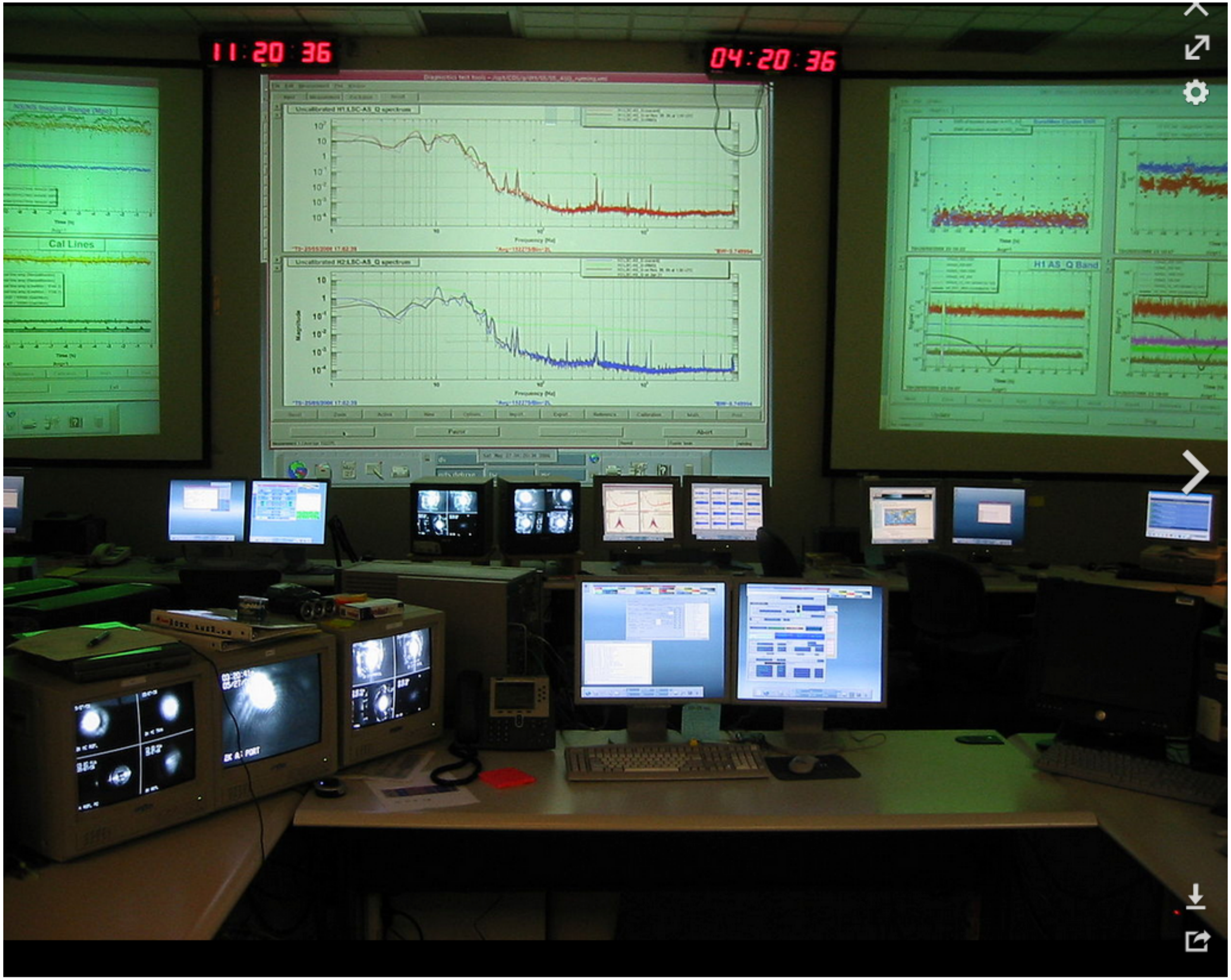
Gravitational wave detection, 2016 [edit]

Main article: Gravitational wave detection

On 11 February 2016, the LIGO collaboration announced the detection of gravitational waves, from a signal detected at 10.51 GMT on 14 September 2015[56] of two black holes with masses of 29 and 36 solar masses merging together around 1.3 billion light years away. The mass of the new black hole obtained from merging the two was 62 solar masses. Energy equivalent to 3 solar masses was emitted as gravitational waves.[57] The signal was seen by both LIGO detectors, in Livingston and Hanford, with a time difference of 7 milliseconds due to the angle between the two detectors and the source. The signal came from the Southern Celestial Hemisphere, in the rough direction of (but much further away than) the Magellanic Clouds.[6] The confidence level of the discovery was 99.99994%.[57]



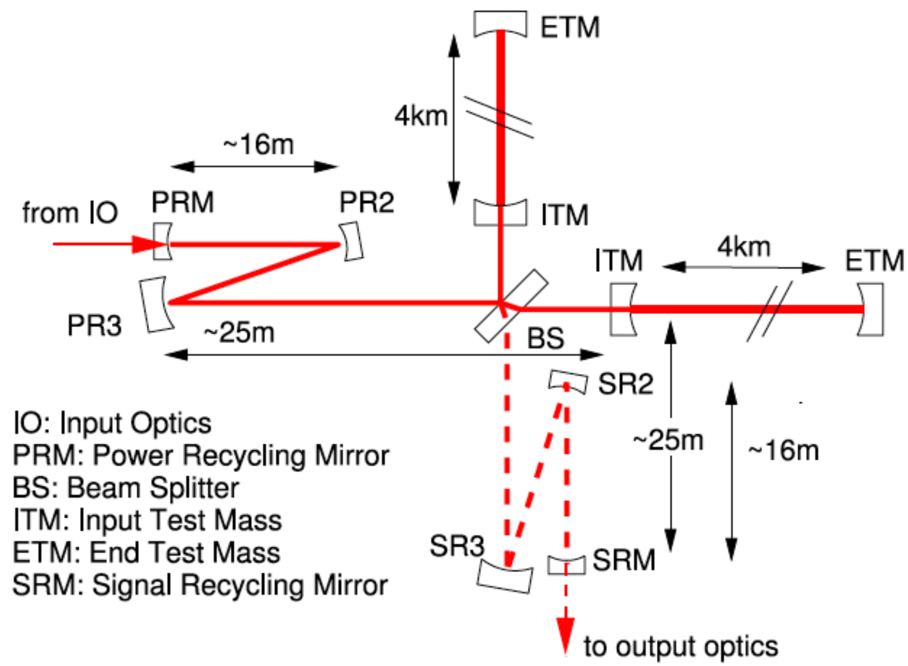
LIGO measurement of the gravitational waves at the Livingston (left) and Hanford (right) detectors, compared to the theoretical predicted values.

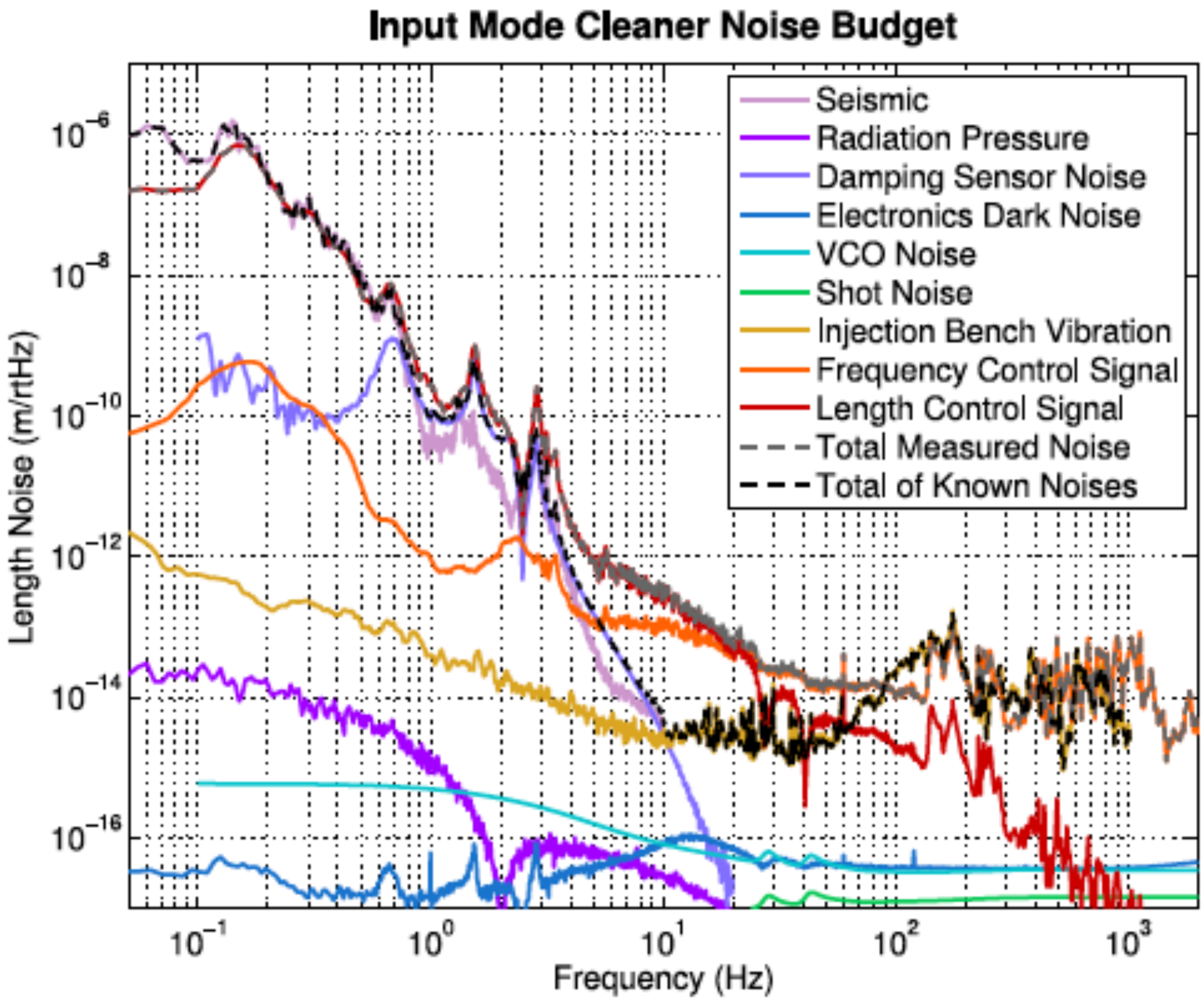


The LIGO Hanford Control Room. Photo taken by Tobin Fricke and uploaded by Philip Neustrom with permission to release under the public domain.

More details

About the Engineering Challenge




Input Mode Cleaner Noise Budget
Length Noise (m/rtHz)
Frequency (Hz)
Seismic
Radiation Pressure
Damping Sensor Noise
Electronics Dark Noise
VCO Noise
Shot Noise
Injection Bench Vibration
Frequency Control Signal
Length Control Signal
Total Measured Noise
Total of Known Noises

Precise Instruments and Control

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Gravitational Waves Detected!

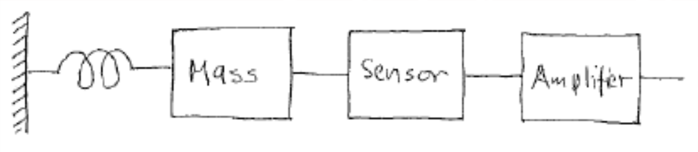
Roughly 1.3 billion years ago, around the time multicellular life was starting to spread on Earth, a pair of black holes collided and released a torrent of gravitational energy into the cosmos. Today, physicists announced they had spotted that energy here on Earth.

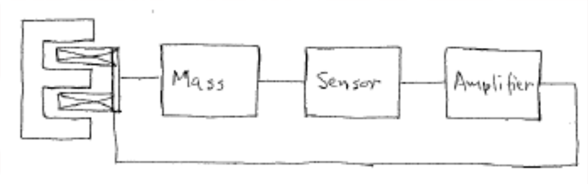
We congratulate the researchers and scientists of LIGO Scientific Collaboration and VIRGO Collaboration for their detection of gravitational waves, confirming Einstein's prediction from 100 years ago! Research published in *Review of Scientific Instruments*, *Physics Today*, *AIP Conference Proceedings*, *American Journal of Physics*, and *Journal of Mathematical Physics* contributed to this discovery. Check out these articles which have been made free to download for a limited time. Physics is awesome! #einsteinwasright

Quest for Detection: Articles from *Review of Scientific Instruments*

Force Feedback

- Classic idea with tremendous impact
- Introduction of actuation and feedback in sensors was a game changer in instrument design





Open loop, all components matter

Bandwidth $\omega_b = \sqrt{k/m}$

Sensitivity = k_a/k

Invariant $\omega_b^2 S = k_a/m$

Closed loop, actuator only critical element

Bandwidth depends on feedback system

Error signal also useful!

Examples from LIGO

- Control of mirror suspensions
- Technique for measurement of quality facto of mechanical oscillators
- Active noise cancellation in a suspended interferometer
- High precision MEMS ground rotation sensor
- State observers and Kalman filtering for vibration systems
- …

Control of mirror suspension

Damping and local control of mirror suspensions for laser interferometric gravitational wave detectors

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Control of mirror suspension

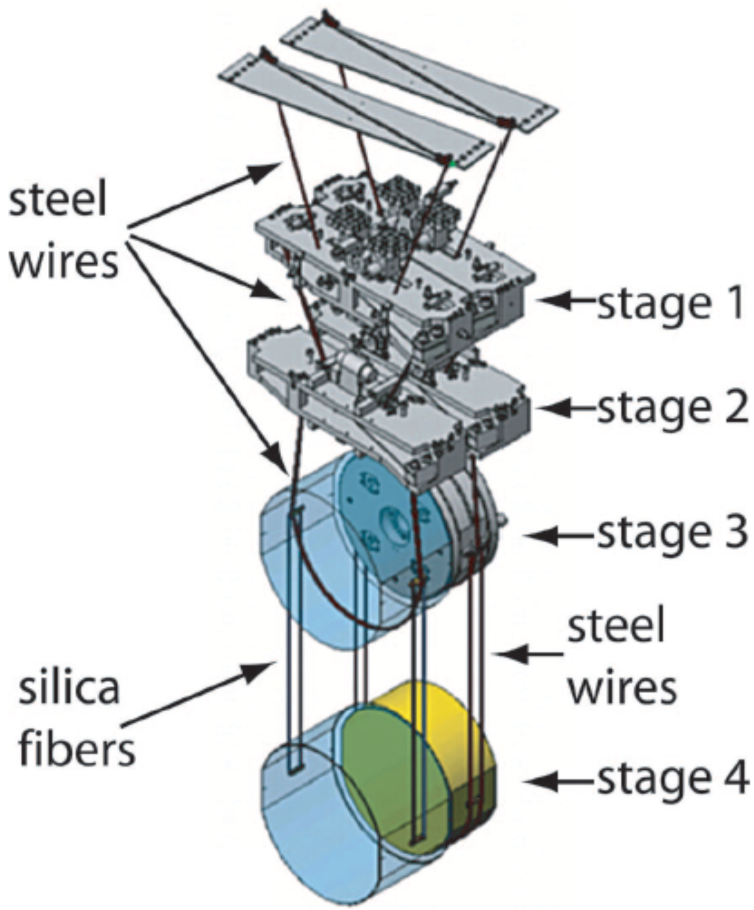


FIG. 1. Modified rendering of a CAD model of the main elements of an aLIGO quadruple suspension. There are two chains of 4 stages, numbered as

Suspension control

II. LOCAL CONTROL OF THE ALIGO QUADRUPLE SUSPENSIONS

The test mass suspension has rigid-body resonances from the lowest x mode at 0.41 Hz to the highest roll mode near 13 Hz. In the region of 0.41 Hz the residual motion of the supporting isolation table is $\sim 2\,\mathrm{nm}/\sqrt{\mathrm{Hz}}$.[8] The mechanical quality factor of the mode is perhaps $Q \sim 10^6$, so the resulting *rms* motion, at $\sqrt{Q}$ times the input spectral density, is of order $2\,\mu\mathrm{m}$. This motion is too large and must be damped, as must most of the other 23 modes.

Requirements

TABLE I. Noise amplitude spectral density limits for the aLIGO test masses. Upper limits are set a factor of 10 below the intended instrumental noise floor, allowing for cross-coupling to the sensitive direction. Each limit falls as $1/f^2$ from 10 Hz to 30 Hz. The interferometer is insensitive to roll, though roll noise can couple into, e.g., x in the mechanical system.

Coordinate	Noise limit at 10 Hz	Units
x	10^{-20}	$\mathrm{m}/\sqrt{\mathrm{Hz}}$
y	10^{-17}	$\mathrm{m}/\sqrt{\mathrm{Hz}}$
z	10^{-17}	$\mathrm{m}/\sqrt{\mathrm{Hz}}$
Yaw	10^{-17}	$\mathrm{rad}/\sqrt{\mathrm{Hz}}$
Pitch	10^{-17}	$\mathrm{rad}/\sqrt{\mathrm{Hz}}$

To allow damping of all stages from the top mass (stage 1) a "marionette"-like arrangement is employed, with the masses coupled so that movement of the top-most stage leads to motion, primarily in the same direction, of all the masses, and vice versa. This is achieved by linking the masses with 4 wires (or fibers), with suitably chosen points of attachment at each mass. In the language of control theory, all the modes to be damped are observable and controllable at the top mass. A fuller description of the mechanical design of the suspension may also be found in Ref. 11.

GOAL: Reduce Q-factor from 10^6 to 10 by active control

Use of simple models

$$\mathbf{M}\ddot{\vec{x}} + \mathbf{K}\vec{x} = \vec{P},$$

$$\vec{x} = \Phi\vec{q},$$

$$\mathbf{M_m}\ddot{\vec{q}} + \mathbf{K_m}\vec{q} = \vec{P}_m.$$

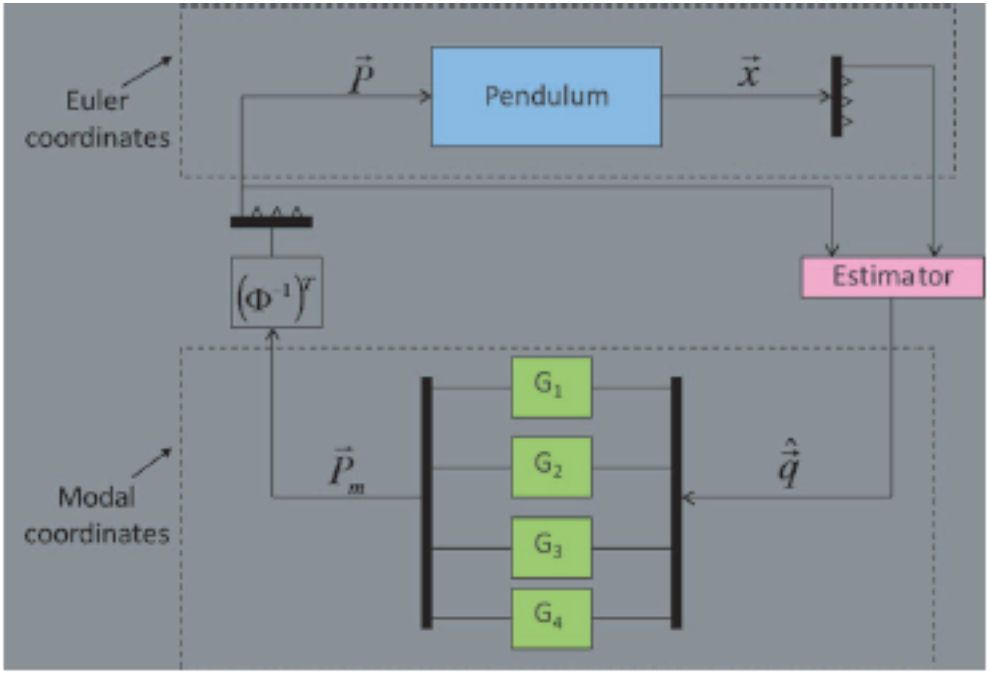


FIG. 2. A block diagram of a modal damping scheme for the 4 x modes. An estimator converts the incomplete sensor information into modal signals. The modal signals are then sent to damping filters, one for each DOF. The resulting modal damping forces are brought back into the Euler coordinate system through the transpose of the inverse of the eigenvector matrix Φ. Only stage 1 forces are applied to maximize sensor noise filtering to stage 4. Note that this figure applies to a four DOF system.

Loop Transfer Function (1Hz mode)

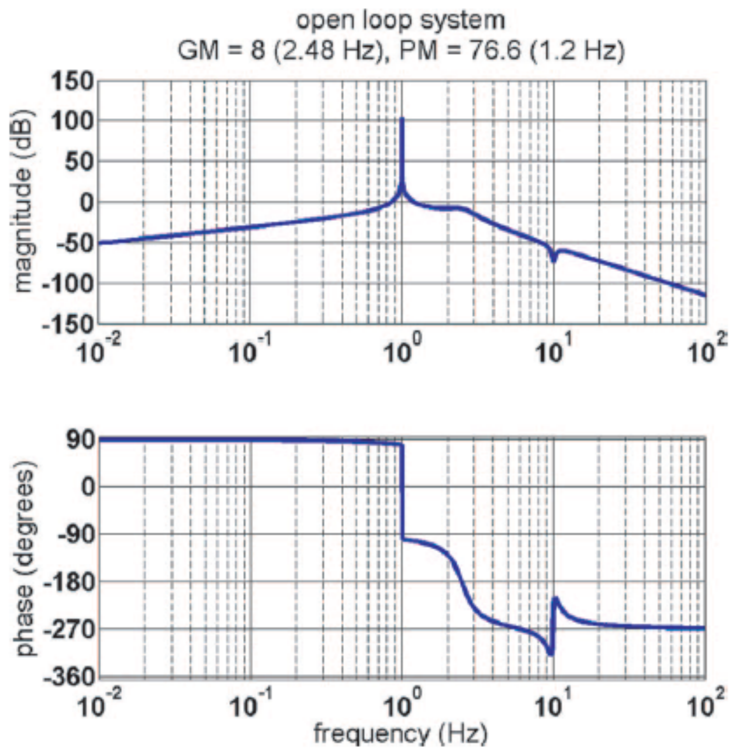
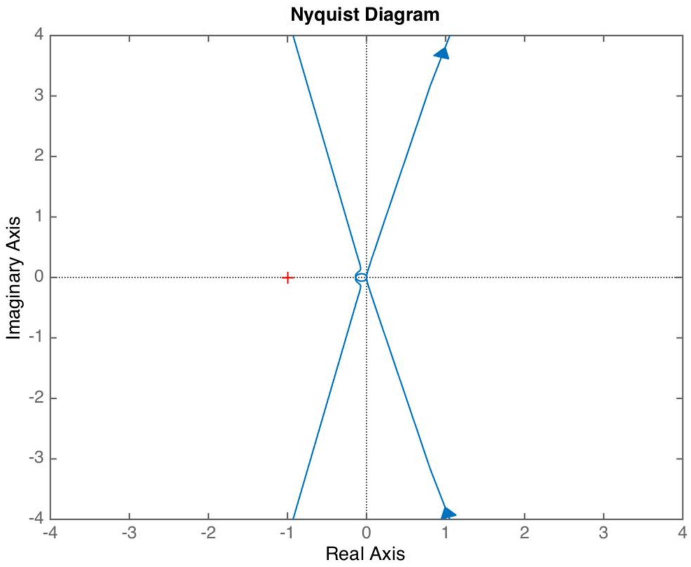
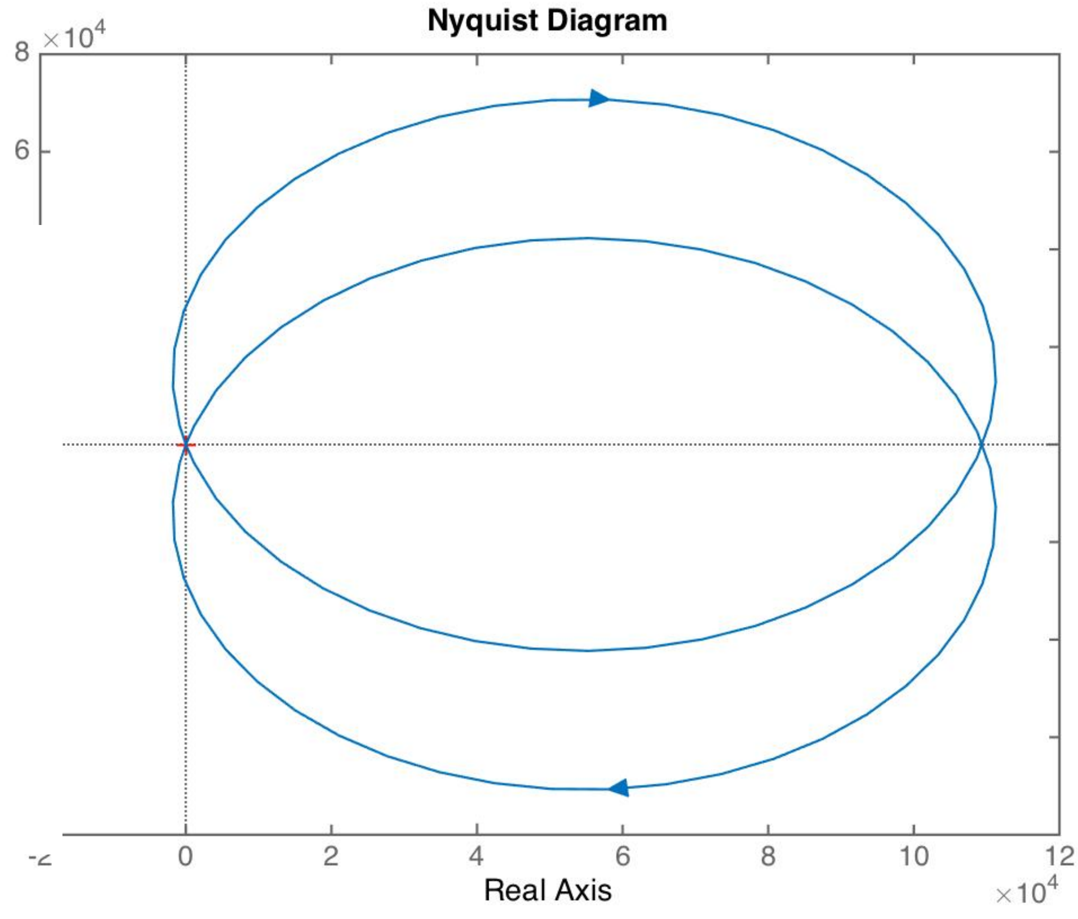


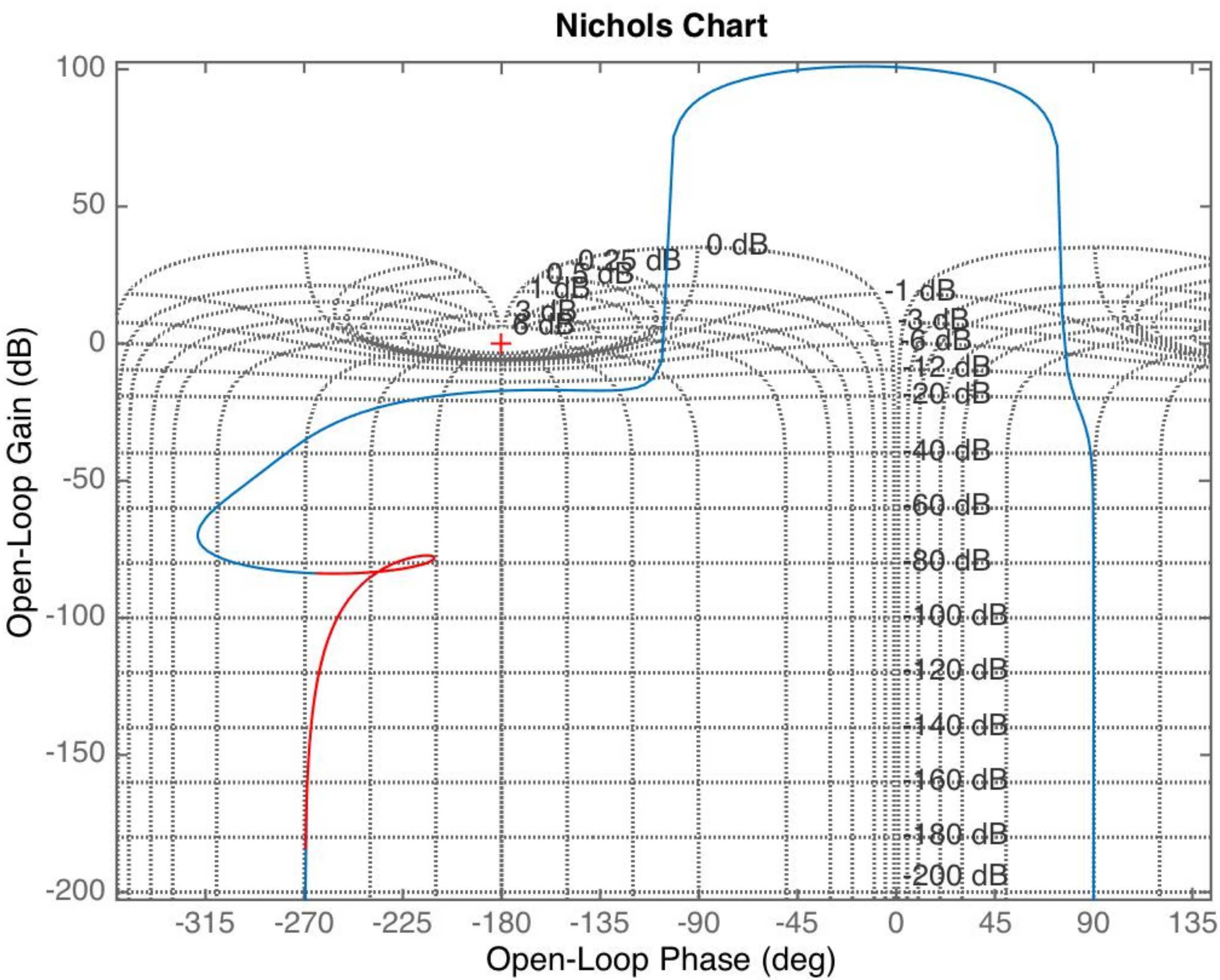
FIG. 3. The loop gain transfer function of an example 1 Hz modal oscillator with its damping filter. The plant contributes the large resonant peak and the damping filter contributes the remaining poles and zeros. The 10 Hz notch reduces the sensor noise amplification at the start of the gravitational wave

$$G_{1\mathrm{Hz}} = \frac{ks(s^2 + 6.283s + 3948)}{(s^2 + 5.455s + 246.7)(s^2 + 62.83s + 3948)}.$$

Nyquist curve







Nichols Chart
Open-Loop Gain (dB)
Open-Loop Phase (deg)
100
50
0
-50
-100
-150
-200
-315
-270
-225
-180
-135
-90
-45
0
45
90
135
0 dB
0.25 dB
0.5 dB
1 dB
3 dB
6 dB
-1 dB
-3 dB
-6 dB
-12 dB
-20 dB
-40 dB
-60 dB
-80 dB
-100 dB
-120 dB
-140 dB
-160 dB
-180 dB
-200 dB

Kalman filter instead of sensor

B. State estimation

The mathematics of modal damping requires the positions of all four stages to be measured. However, only stage 1 is directly observed, as Figure 2 illustrates. The stages below have effectively no sensors since any measurement would refer a moving platform with its own dynamics. Consequently an estimator, as in the equation,

$$\begin{bmatrix} \dot{\hat{\vec{q}}} \\ \ddot{\hat{\vec{q}}} \end{bmatrix} = \mathbf{A}_m \begin{bmatrix} \hat{\vec{q}} \\ \dot{\hat{\vec{q}}} \end{bmatrix} + \mathbf{B}_m \vec{u} - \mathbf{L}_m \left(\mathbf{C}_m \begin{bmatrix} \hat{\vec{q}} \\ \dot{\hat{\vec{q}}} \end{bmatrix} - \vec{y} \right), \qquad (8)$$

must be employed to reconstruct the full dynamics.

The estimator is designed using the Linear Quadratic Regulator (LQR) technique which solves the cost function in Eqs. (9) and (10):

$$J = \int_0^\infty \left([\,\tilde{\vec{q}}^T \quad \dot{\tilde{\vec{q}}}^T\,] \mathbf{Q} \begin{bmatrix} \tilde{\vec{q}} \\ \dot{\tilde{\vec{q}}} \end{bmatrix} + \vec{z}_m^T \mathbf{R} \vec{z}_m \right) dt, \tag{9}$$

$$\text{with } \mathbf{L}_m = \underset{\mathbf{L}_m}{\operatorname{argmin}}(J). \tag{10}$$

The $\mathbf{Q}$ matrix weights the accuracy of the modal state estimation while the $\mathbf{R}$ matrix weights the cost of using a noisy measurement. Here $\vec{z}_m$ is defined as

$$\vec{z}_m = -\mathbf{L}_m^T \begin{bmatrix} \tilde{\vec{q}} \\ \dot{\tilde{\vec{q}}} \end{bmatrix}, \tag{11}$$

LQG Tuning

First, **Q** is set by placing the square of the inverse of the resonance frequencies on the diagonal. In this way lower resonance frequencies, which have more mechanical energy, will be damped more efficiently. Only the value of **R** remains to be optimized.

To choose the best value of **R** the control design is first set so that the damping time requirement is met assuming full state information. An optimization routine in MATLAB® then simulates the performance of the closed loop system for estimators designed using many values of **R** over a sufficiently large space. A cost function, Eq. (12), dependent on the performance criteria is calculated for each value of **R**. The value

LQG Tuning

The two competing performance criteria of damping time and technical noise are stated above. The sensor noise is a known measured quantity approximately $7 \times 10^{-11}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$ beyond 10 Hz. Since the contribution to mirror motion drops off quickly in the frequency domain, we will only consider the contribution at 10 Hz, the start of the gravitational wave detection band.

These two performance criteria are represented in the optimization routine with the cost function:

$$J_R(\mathbf{R}) = \max_{\mathrm{DOF}}(T_s^2) + \max_{\mathrm{DOF}}(N^2), \qquad (12)$$

$$\text{with } \mathbf{R} = \operatorname*{argmin}_{\mathbf{R}}(J_R). \qquad (13)$$

Results

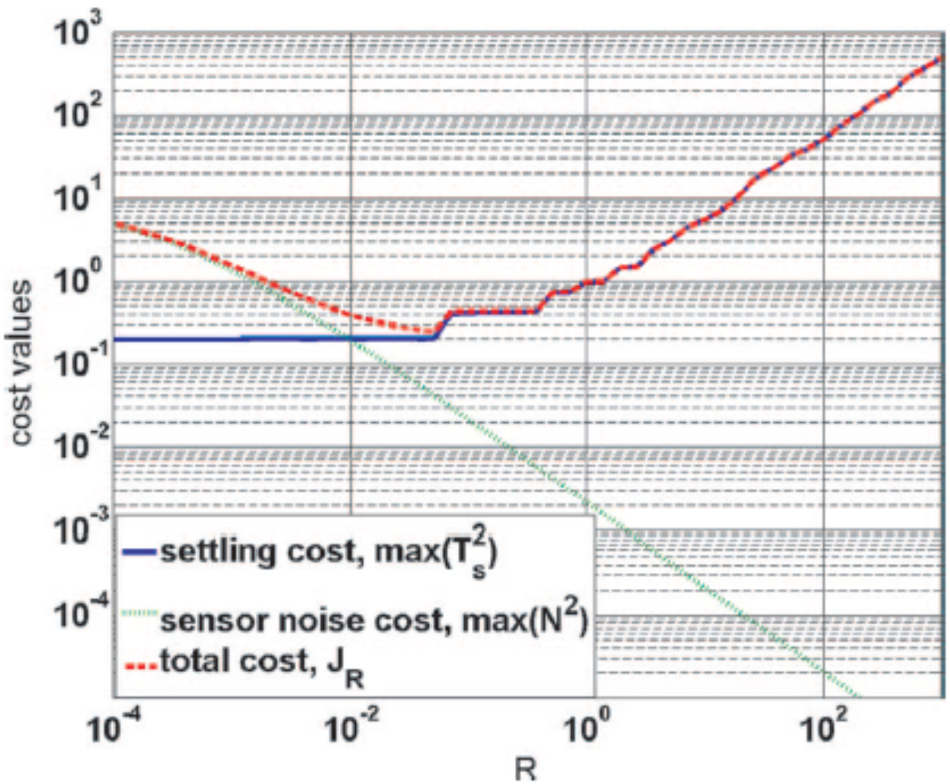


FIG. 4. The components of the cost function Eq. (12) for the x DOF as a function of **R** calculated by the optimization routine. At each value of **R** the closed loop system performance is simulated using the estimator design based on the LQR solution with that particular **R** value.

Evaluating Robustnes

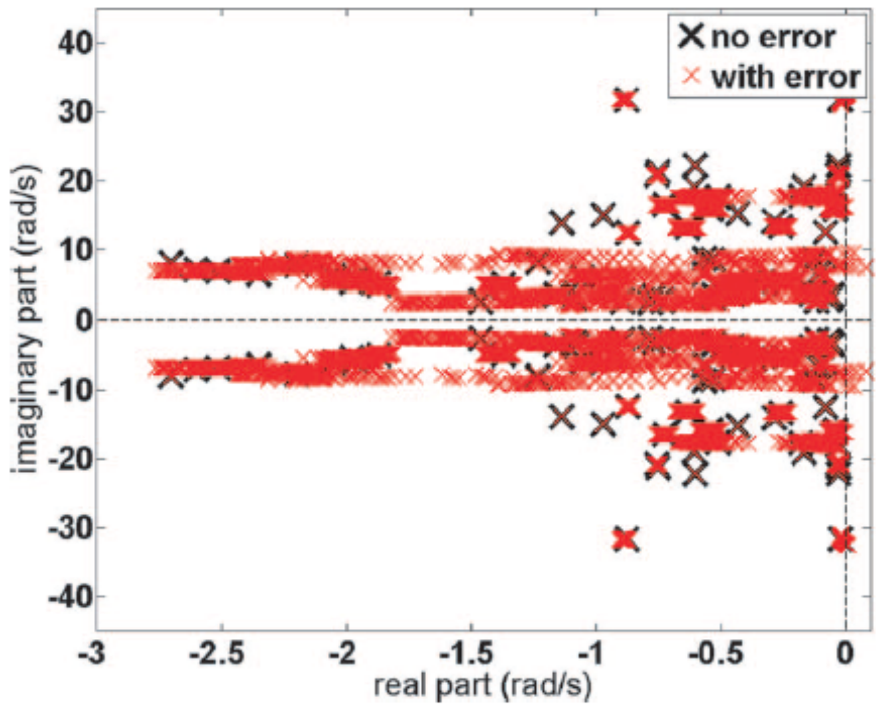


FIG. 6. Complex frequency (s)-plane plot showing the poles $\times$ of the closed loop modal damping system. The reference system is represented by the bold (black) symbols, while the 100 trials of perturbed systems are represented by the finer (red) symbols. Each trial represents a system modified from the ideal using the random parameters described in the text. In this test 16% of the cases are unstable.

Summary

We have presented the rationale and design method for active damping of the mirror-suspensions of a gravitational wave detector and shown that a combination of approaches can be expected to provide the required performance. A particular advantage of our approach is that relatively simple sensors are adequate to meet even the exceedingly tight noise tolerances associated with gravitational wave detection.