PID as a Benchmark for Event-Based Control?

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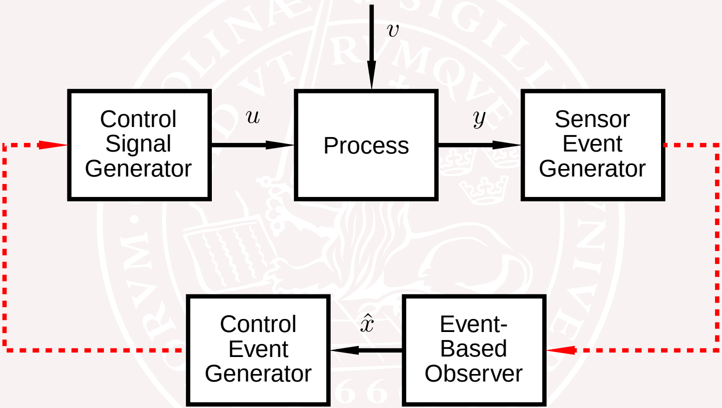
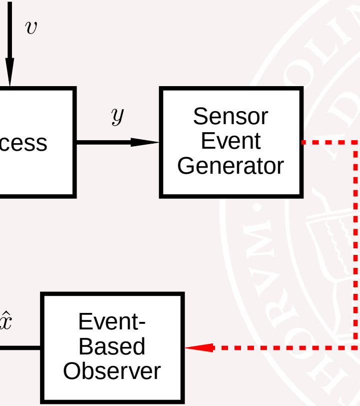
Event-Based Control



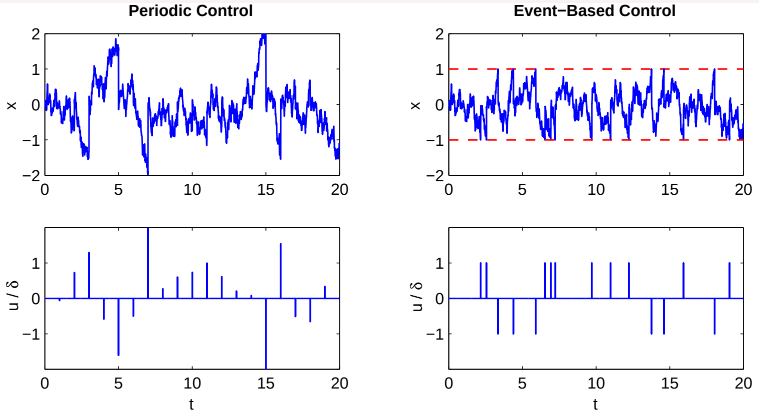
Event-Based Control



Sensor event generators:

- Periodic sampler, interval h
- Send-on-Delta:
  sample if $|y - y_{\text{last}}| > \Delta$
- Stochastic Send-on-Delta:
  sample with probability
  $p = 1 - e^{-\frac{(y-y_{\text{last}})^2}{2\sigma_s^2}}$

Example: Impulse Control of a Wiener Process



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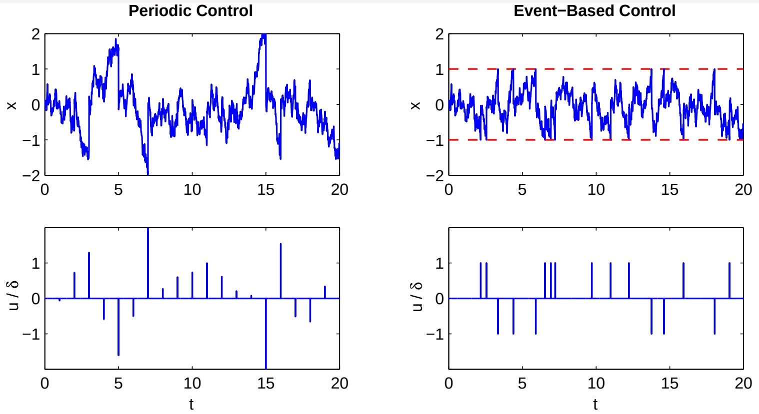

Periodic sampling: $\mathrm{E}\ x^2 = \frac{1}{2}h$

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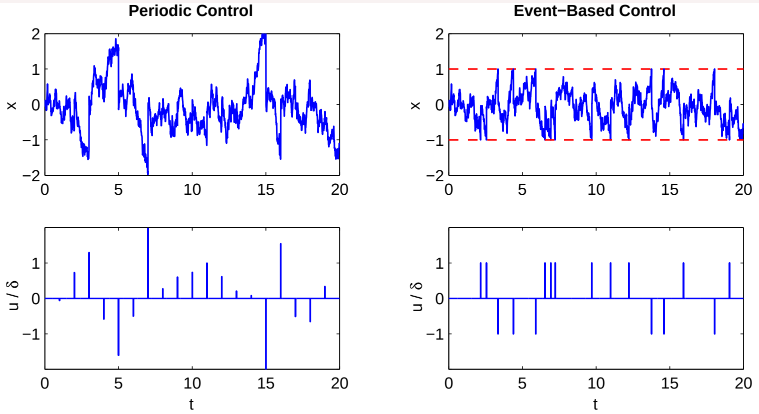

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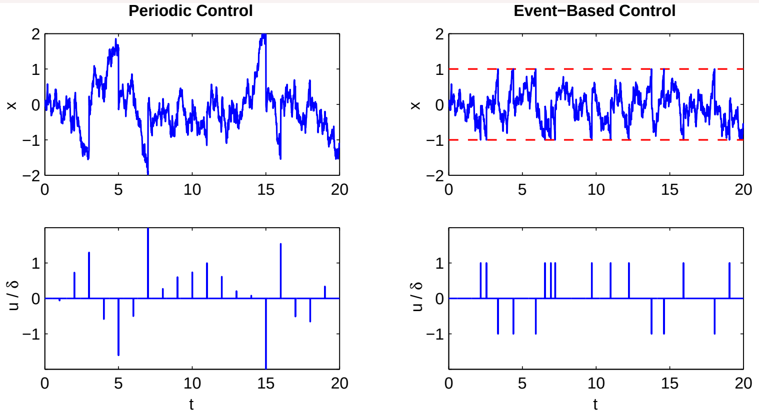

Periodic sampling: $\mathrm{E}\, x^2 = \frac{1}{2} h$

Send-on-Delta: $\mathrm{E}\, x^2 = \frac{1}{6} \bar{h}$

Stochastic Send-on-Delta: $\mathrm{E}\, x^2 \approx 0.291 \bar{h}$

Event-Based Control

Two separate lines of research:

1. Optimal estimation and control of stochastic systems
   - Karl Johan Åström & Bo Bernhardsson (1999)
   - Toivo Henningsson (2012)
   - . . .
2. Heuristic event-based PID control
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Can we somehow compare these apples and oranges?

- Can we use PID control as a benchmark for event-based estimation and control?
- Can we evaluate heuristic event-based PID controllers using metrics from stochastic control?

Benchmark Design

We would like to have a benchmark were we can compare

- continuous control
- sampled-data control
- event-based control (various versions)

with respect to

- performance
  - disturbance rejection
  - control effort
  - number of sensor and control events
- design complexity
- implementation complexity and computational effort

Benchmark Design

It would be nice to have

- known, optimal solutions for continuous and sampled-data control
- the possibility to evaluate the performance analytically or using Monte Carlo simulations

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LQG-Optimal PI(D) Control

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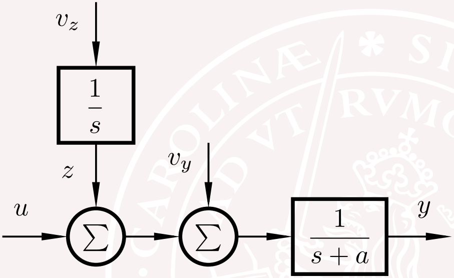
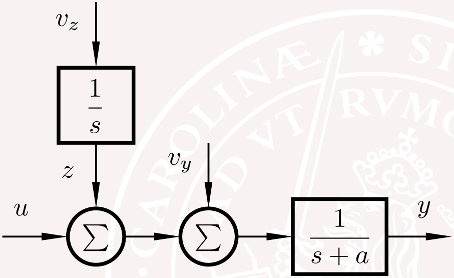


v_z, v_y white noise proc. with intensities r and 1

LQG-Optimal PI Control



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Cost function:

$$J = \mathrm{E}\left\{qy^2 + (u+z)^2\right\}$$

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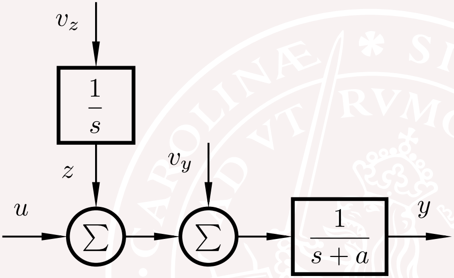

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State feedback: $u = -ly - \hat{z}$

$$l = a + \sqrt{a^2 + q}$$

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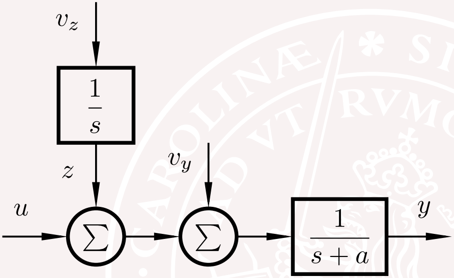

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State feedback: $u = -ly - \hat{z}$ $\qquad l = a + \sqrt{a^2 + q}$

Kalman filter: $\dot{\hat{z}} = k(\dot{y} + ay - \hat{z} - u)$ $\qquad k = \sqrt{r}$

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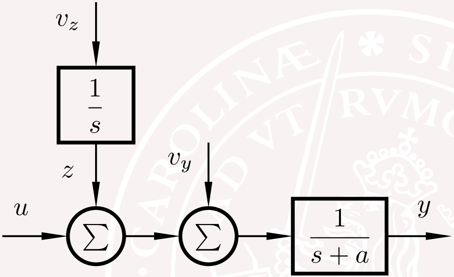

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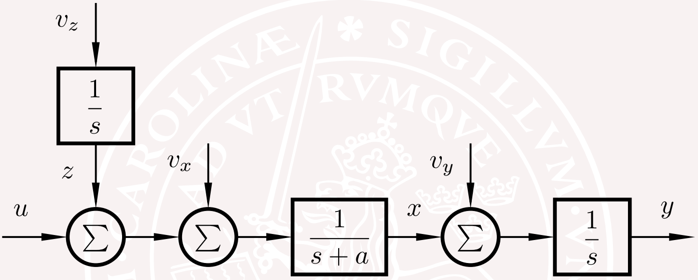
Controller: $U(s) = -\dfrac{(l+k)s + k(l+a)}{s} Y(s)$ (PI)

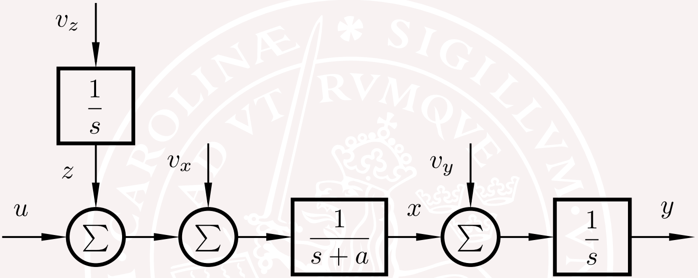
$$(q, r) \Leftrightarrow (K, T_i)$$

LQG-Optimal PID Control



v_z, v_x, v_y white noise processes with intensities r_z and r_x and 1.

LQG-Optimal PID Control



v_z, v_x, v_y white noise processes with intensities r_z and r_x and 1.

Cost function: $J = \mathrm{E}\left\{q_y y^2 + q_x x^2 + (u+z)^2\right\}$

LQG-Optimal PID Control

State feedback: $u = -l_y y - l_x \hat{x} - \hat{z}$

$$l_y = \sqrt{q_y}$$
$$l_x = a + \sqrt{a^2 + 2\sqrt{q_y} + q_x}$$

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State feedback: $u = -l_y y - l_x \hat{x} - \hat{z}$

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Kalman filter:
$$\dot{\hat{z}} = k_z(\dot{y} - \hat{z})$$
$$\dot{\hat{x}} = a\hat{x} + \hat{z} + u + k_x(\dot{y} - \hat{z})$$

$$k_z = \sqrt{r_z}$$
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$$k_z = \sqrt{r_z}$$
$$k_x = \sqrt{2\sqrt{r_z} + r_x}$$

Controller:

$$U(s) = -\frac{(k_z + l_y + k_x l_x)s^2 + (l_y(k_x - a) + k_z(l_x - a))s + k_z l_y}{s^2 + (k_x + l_x - a)s} Y(s)$$

(PID with first-order filter)

$$(q_y, q_x, r_z, r_x) \Leftrightarrow (K, T_i, T_d, N)$$

Benchmark 1 – PI

PI control of an integrator with integral disturbance

Let $r = q = 1 \;\Rightarrow\; K = 2, T_i = 2$

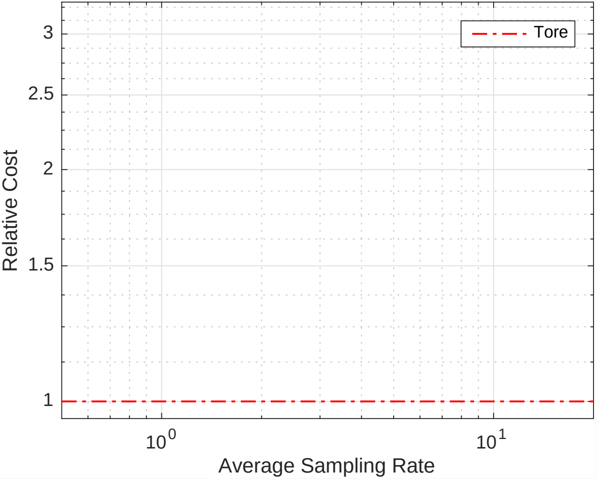
Compare:

- Continuous-time PI control
- (Optimal) sampled-data PI control
- Send-on-Delta + Toivo's Bayesian event-based observer
- Send-on-Delta + Karl-Erik's simple event-based PI controller
- Stochastic Send-on-Delta + time-varying Kalman filter

Karl-Erik's Simple Event-Based PID Controller

```
(* Pre-calculated parameter *)
bi := K / Ti;
(* Event detection *)
ysp := ADIn(ch1);
y := ADIn(ch2);
e := ysp - y;
hact := hact + hnom;
IF (abs(e - es) > elim) OR (hact >= hmax) THEN
  es := e;
  ad := Td/(Td + N*hact);
  (* Calculate control signal *)
  up := K*(beta * ysp - y);
  ud := ad*ud - ad*K*N*(y - yold);
  u := up + ui + ud;
  DAOut(u,ch3);
  (* Update states *)
  ui := ui + bi*hact*(ysp - y);
  yold := y;
  hact := 0.0;
ENDIF;
```

Benchmark 1 – PI – Results



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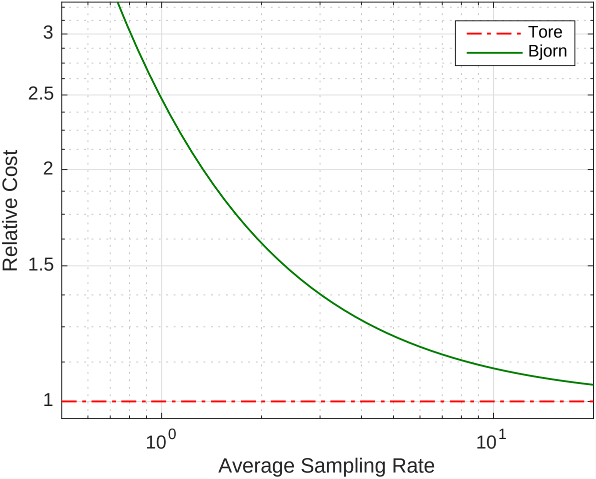

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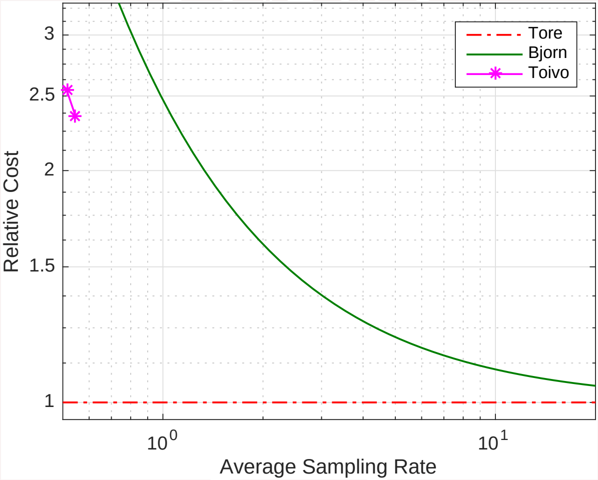

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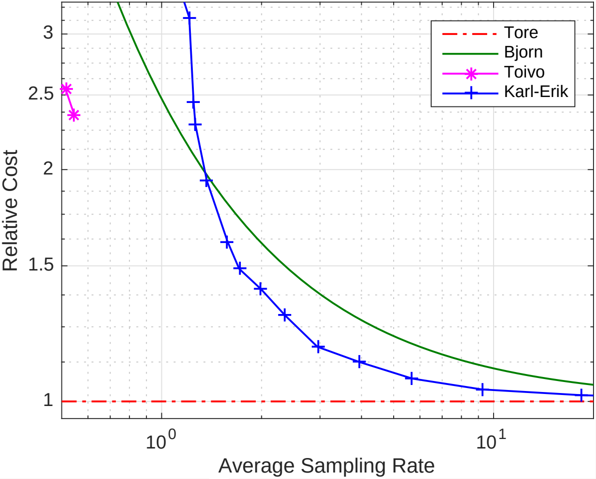

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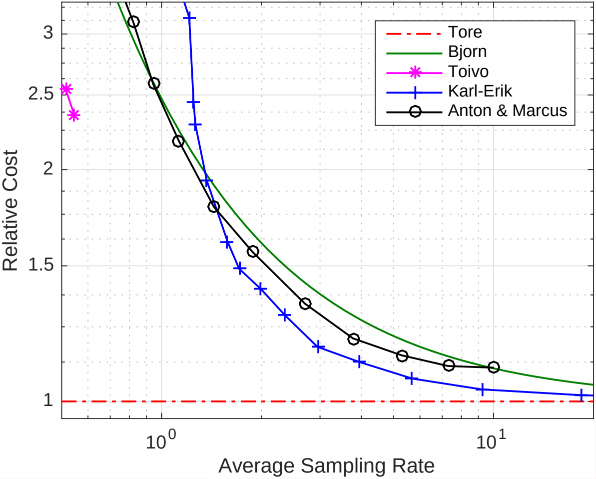


Benchmark Example 2

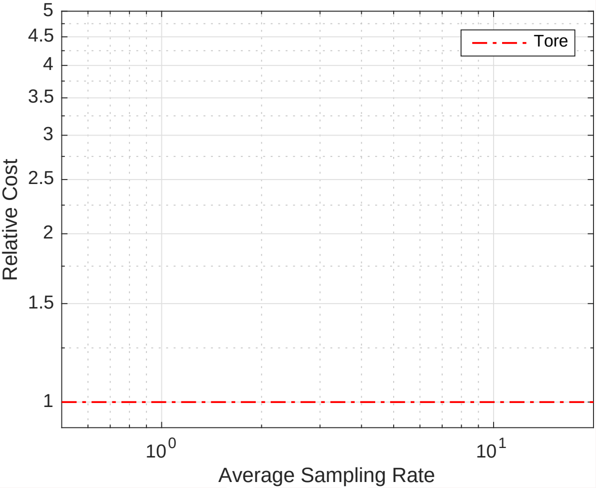
PID control of a double integrator with integral disturbance

$$q_y = q_x = r_z = r_x = 1 \;\Rightarrow\; K = 0.92, T_i = 3.2, T_d = 1.3, N = 4.4$$

Compare:

- Continuous-time PID control
- (Optimal) sampled-data PID control
- Send-on-Delta + Karl-Erik's simple event-based PID controller
- Stochastic Send-on-Delta + time-varying Kalman filter

Benchmark 2 (PID) – Results



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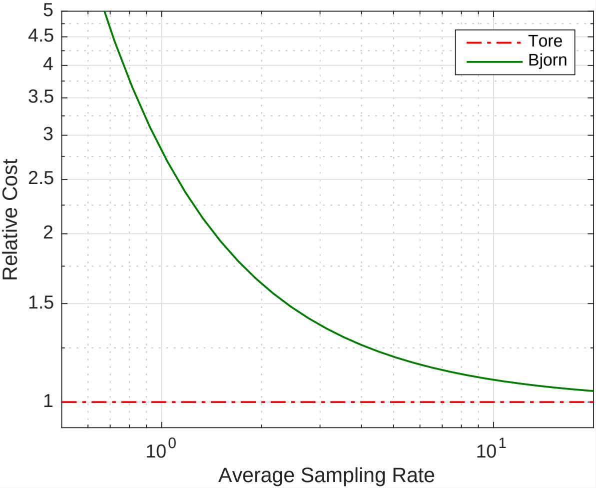

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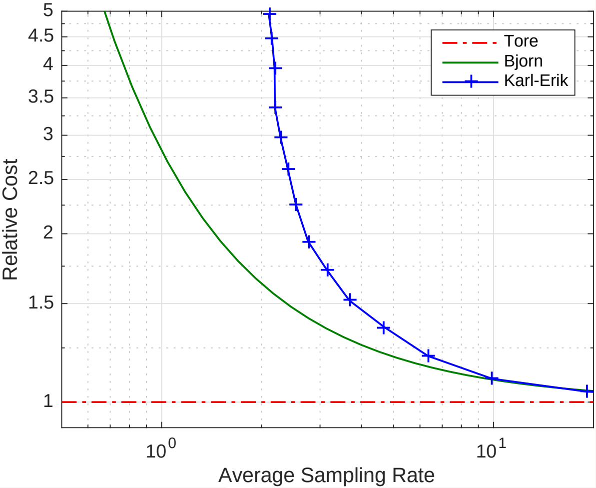

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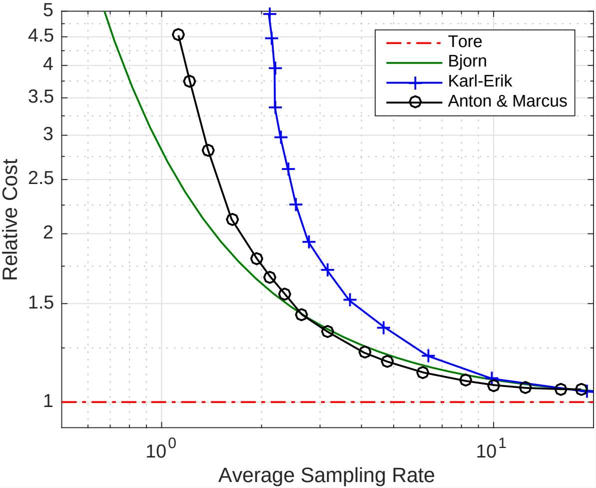

Conclusions and Discussion

- A good state estimation algorithm does not necessarily imply good closed-loop control
  - Design of sensor and control generators
  - Dual control effects
- Constant-intensity white noise models do not favor event-based control
  - Do more suitable stochastic models exist, e.g., intermittent disturbances?
  - Implications for control design?
- Can standard PID metrics be used and can they be evaluated experimentally?
  - $IAE_{\mathrm{load}}, M_s, \ldots$