Least-costly application-oriented input design

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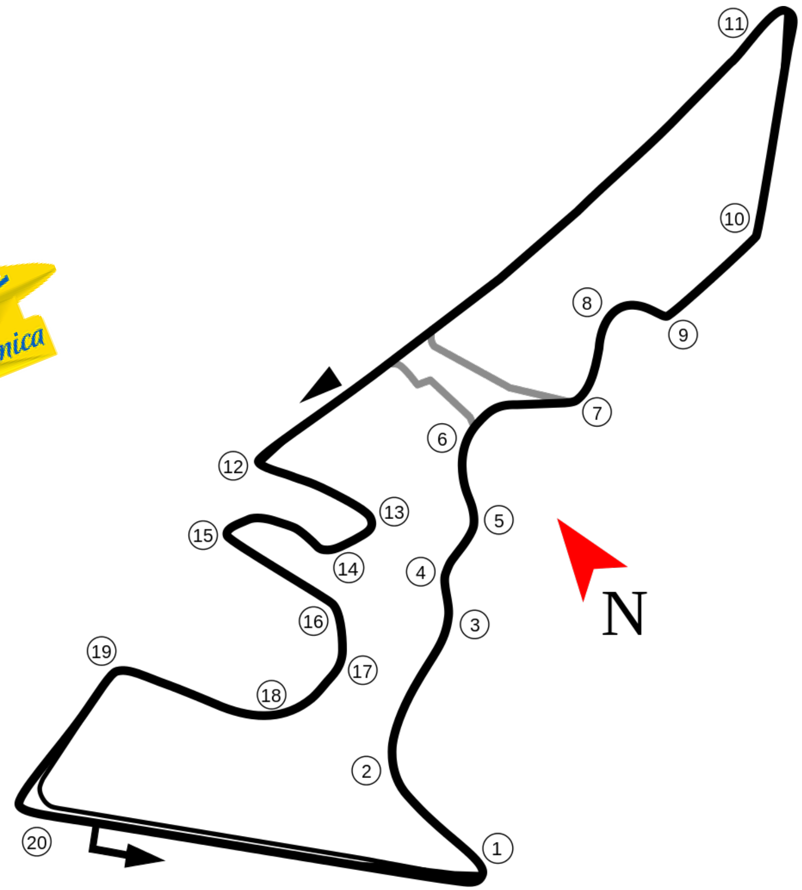
Motivation



3 billion SEK/year

Motivation

3 billion SEK/year

Motivation

30 000 SEK/year

Motivation

“If the ultimate purpose is to design a control system then it seems logical that the accuracy of an identification should be judged on the basis of the performance of the control system designed from the results of the identification.”

Åström & Eykhoff (1971)

Motivation

“It is estimated that 75% of the cost related to a control project in industry is dedicated to the identification of a model.”

Hussain (1999)

Introduction

Framework for experiment design in system identification for control.

- Objective:

  Find optimal input signal to be used in system identification experiment.

- Such that:

  The control application specification is guaranteed when using the estimated model in the control design.

Notation

The model structure is parametrized by θ.

- True system is given by θ_0.
- Estimated model is given by $\hat{\theta}$.

Application set

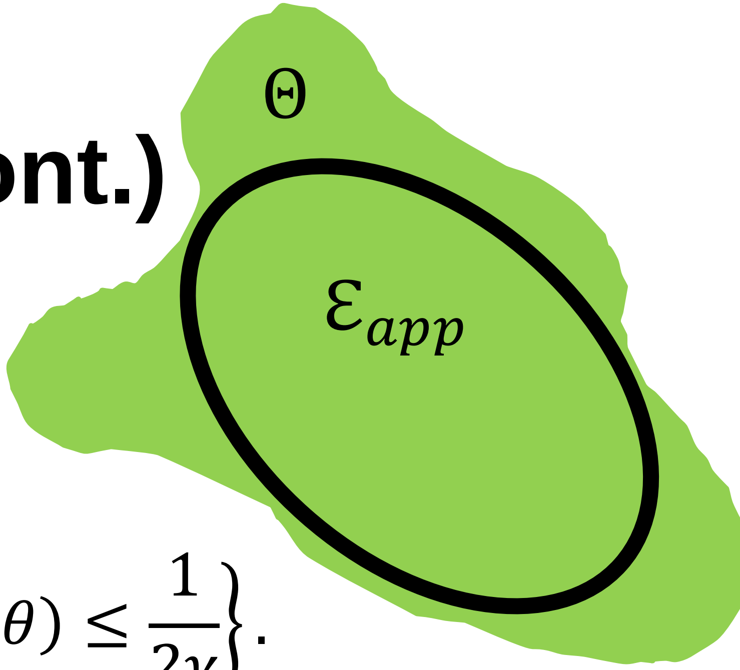
- Application cost: $V_{app}(\theta)$ such that

$$V_{app}(\theta_0) = 0, V'_{app}(\theta_0) = 0, V''_{app}(\theta_0) \succcurlyeq 0.$$

- Application specification

$$V_{app}(\theta) \leq \frac{1}{2\gamma}, \gamma > 0.$$

Application set (cont.)



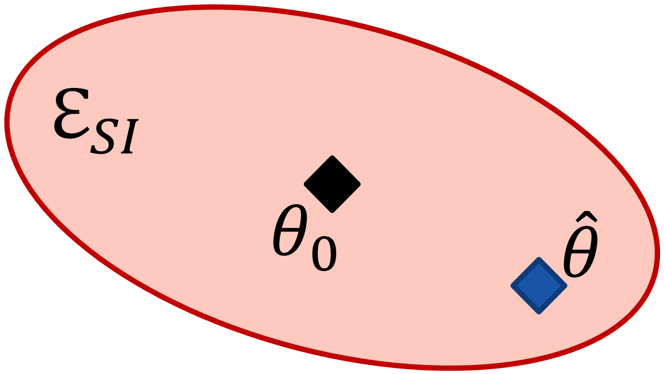
- Acceptable parameter set

$$\Theta(\gamma) = \left\{\theta \,\middle|\, V_{app}(\theta) \leq \frac{1}{2\gamma}\right\}.$$

- Ellipsoidal approximation

$$\Theta(\gamma) \approx \mathcal{E}_{app}(\gamma) = \left\{\theta \,\middle|\, (\theta - \theta_0)^T V''_{app}(\theta)(\theta - \theta_0) \leq \frac{1}{\gamma}\right\}.$$

System identification set



Asymptotic property

$$\hat{\theta} \in \mathcal{E}_{SI}(\eta) = \{\theta | (\theta - \theta_0)^T I_F (\theta - \theta_0) \leq \eta\}.$$

(Key result from prediction error/maximum likelihood system identification.)

Optimal input design

- Estimated parameters:

$$\hat{\theta} \in \mathcal{E}_{SI}(\eta) = \{\theta | (\theta - \theta_0)^T I_F (\theta - \theta_0) \leq \eta\}.$$

- Acceptable parameters in application:

$$\hat{\theta} \in \Theta(\gamma) \approx \mathcal{E}_{app}(\gamma) = \left\{\theta \middle| (\theta - \theta_0)^T V''_{app}(\theta)(\theta - \theta_0) \leq \frac{1}{\gamma}\right\}.$$

- Experiment cost:

$$f_{cost}\left(\Phi_u, \Phi_y\right).$$

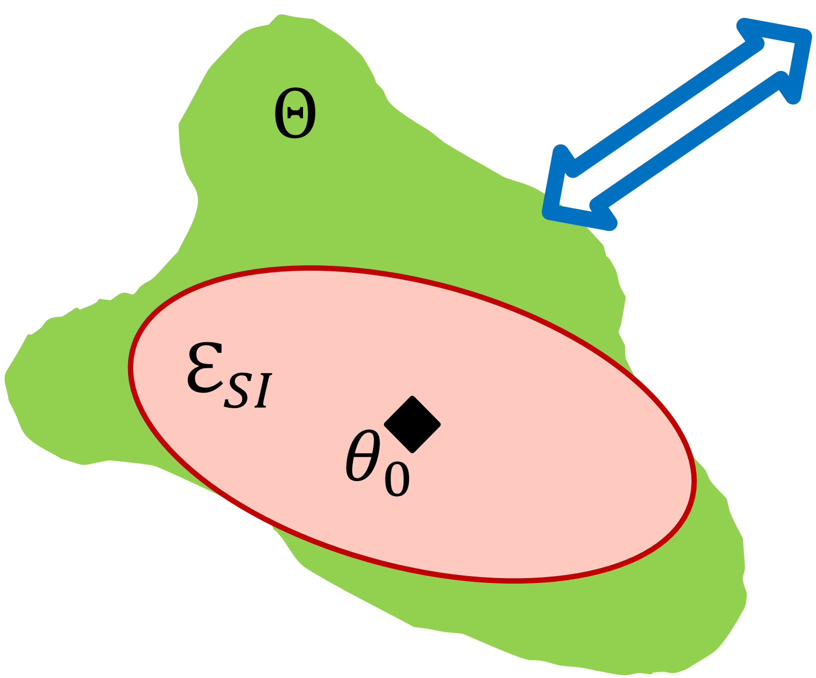
Optimal input design (cont.)

$$\underset{\Phi_u, \Phi_y}{\text{minimize}} \; f_{cost}(\Phi_u, \Phi_y),$$

$$\text{subject to} \quad \mathcal{E}_{SI}(\eta) \subseteq \Theta(\gamma),$$

$$0 \leq \Phi_u(\omega), \forall \omega,$$

$$0 \leq \Phi_y(\omega), \forall \omega.$$



Optimal input design (cont.)

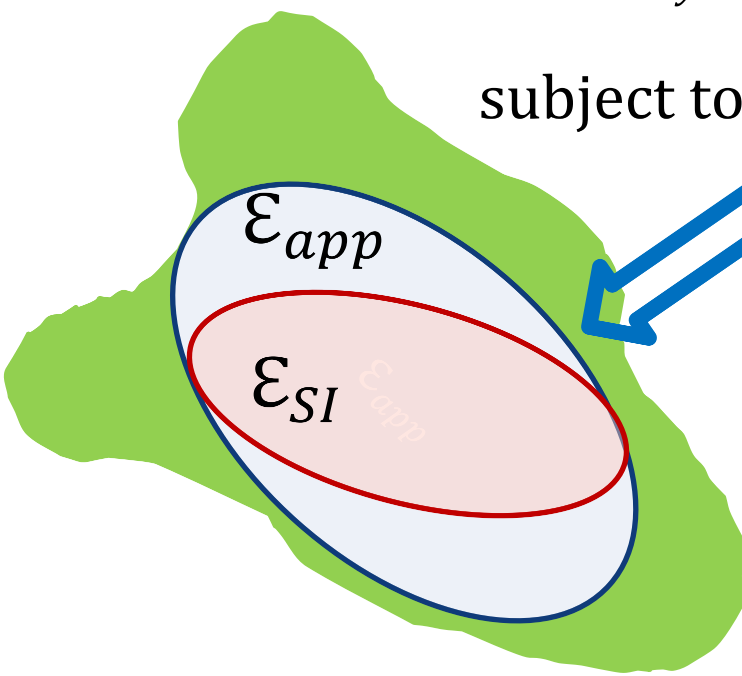
Approximative problem formulation

$$\underset{\Phi_u, \Phi_y}{\text{minimize}} \quad f_{cost}(\Phi_u, \Phi_y),$$

$$\text{subject to} \quad I_F \succcurlyeq \eta\gamma V''_{app}(\theta_0)$$

$$0 \leq \Phi_u(\omega), \forall \omega$$

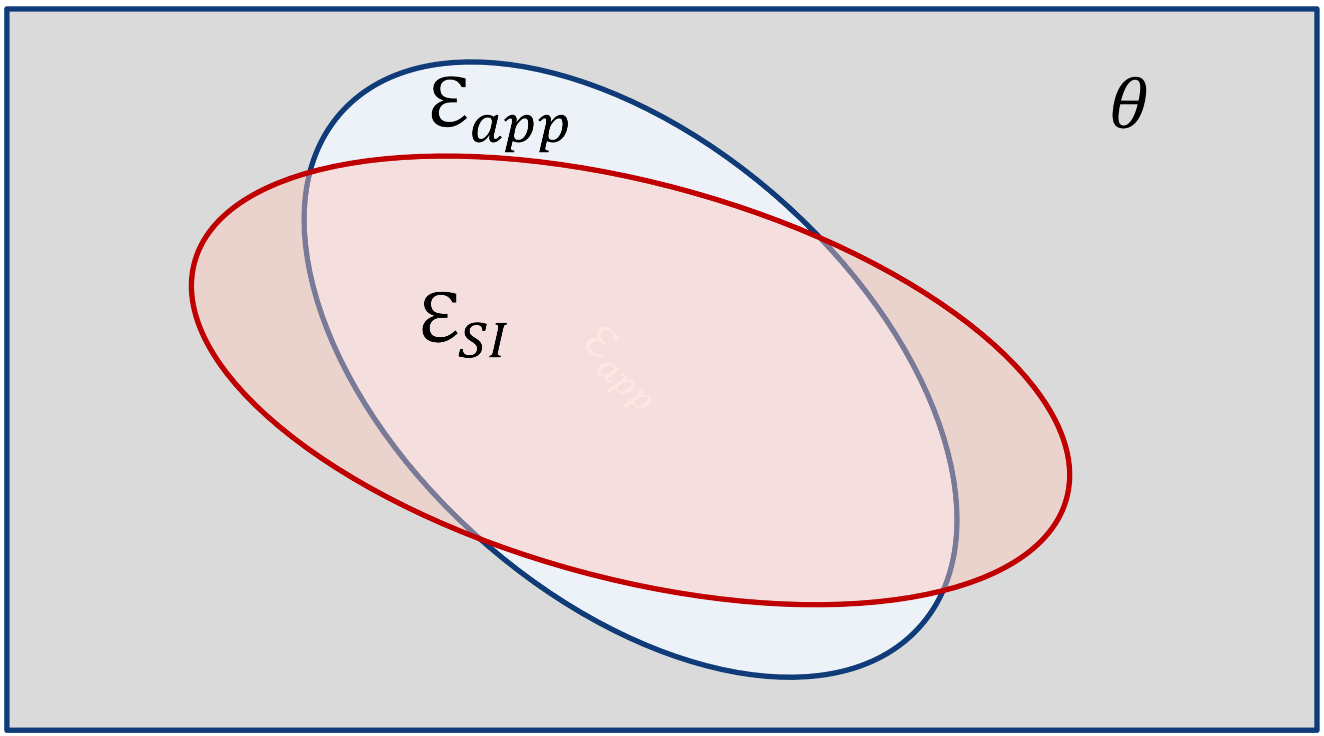
$$0 \leq \Phi_y(\omega), \forall \omega$$



convex problem

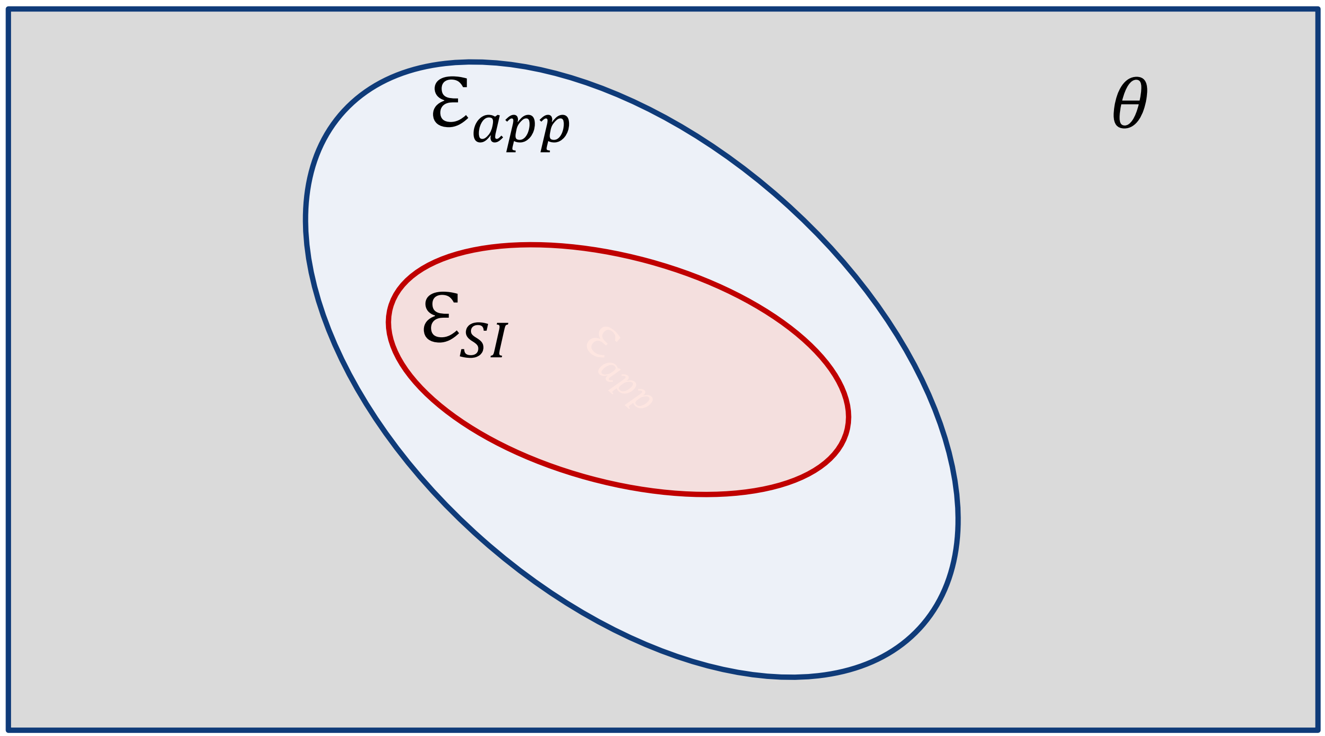
Optimal input design (cont.)

Geometric intepretation



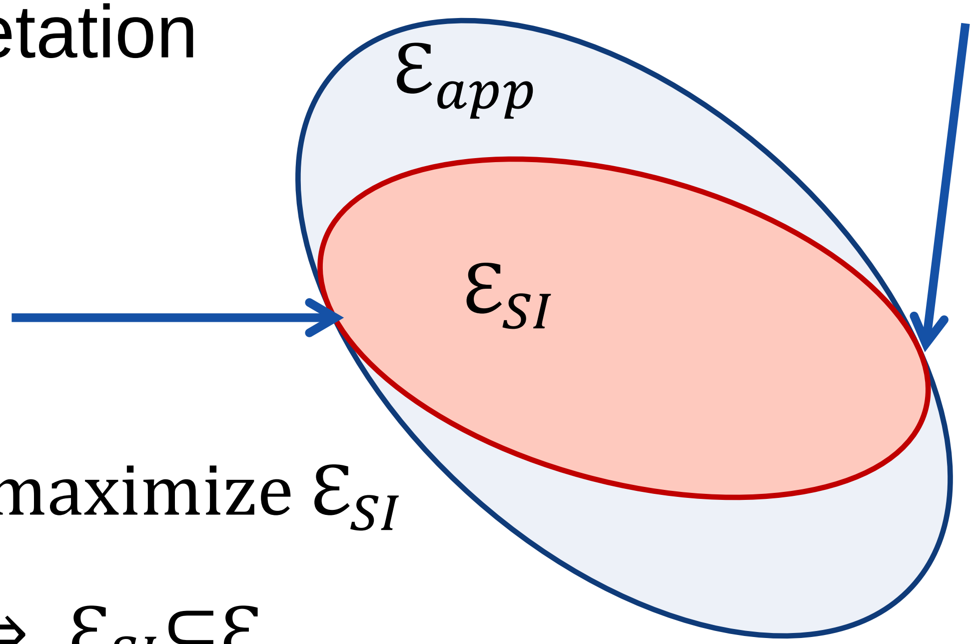
Optimal input design (cont.)

Geometric intepretation



Optimal input design (cont.)

Geometric intepretation

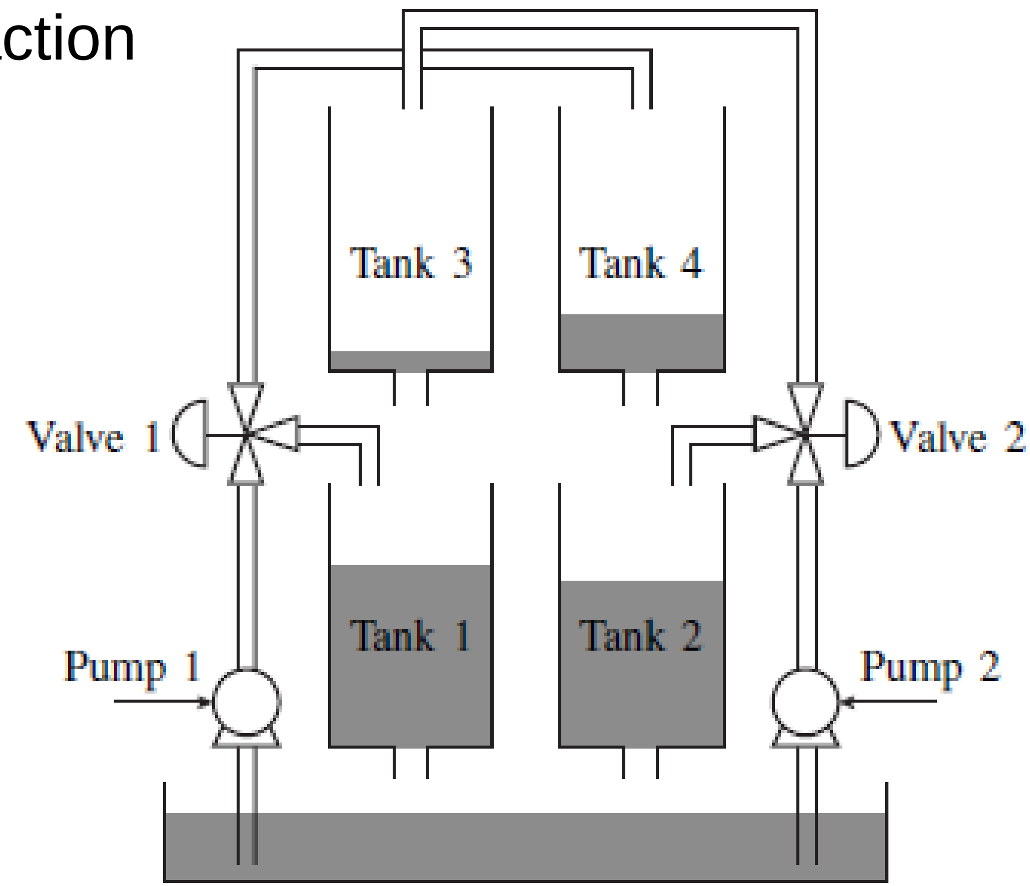


$$\text{minimize } f_{cost} \Leftrightarrow \text{maximize } \mathcal{E}_{SI}$$

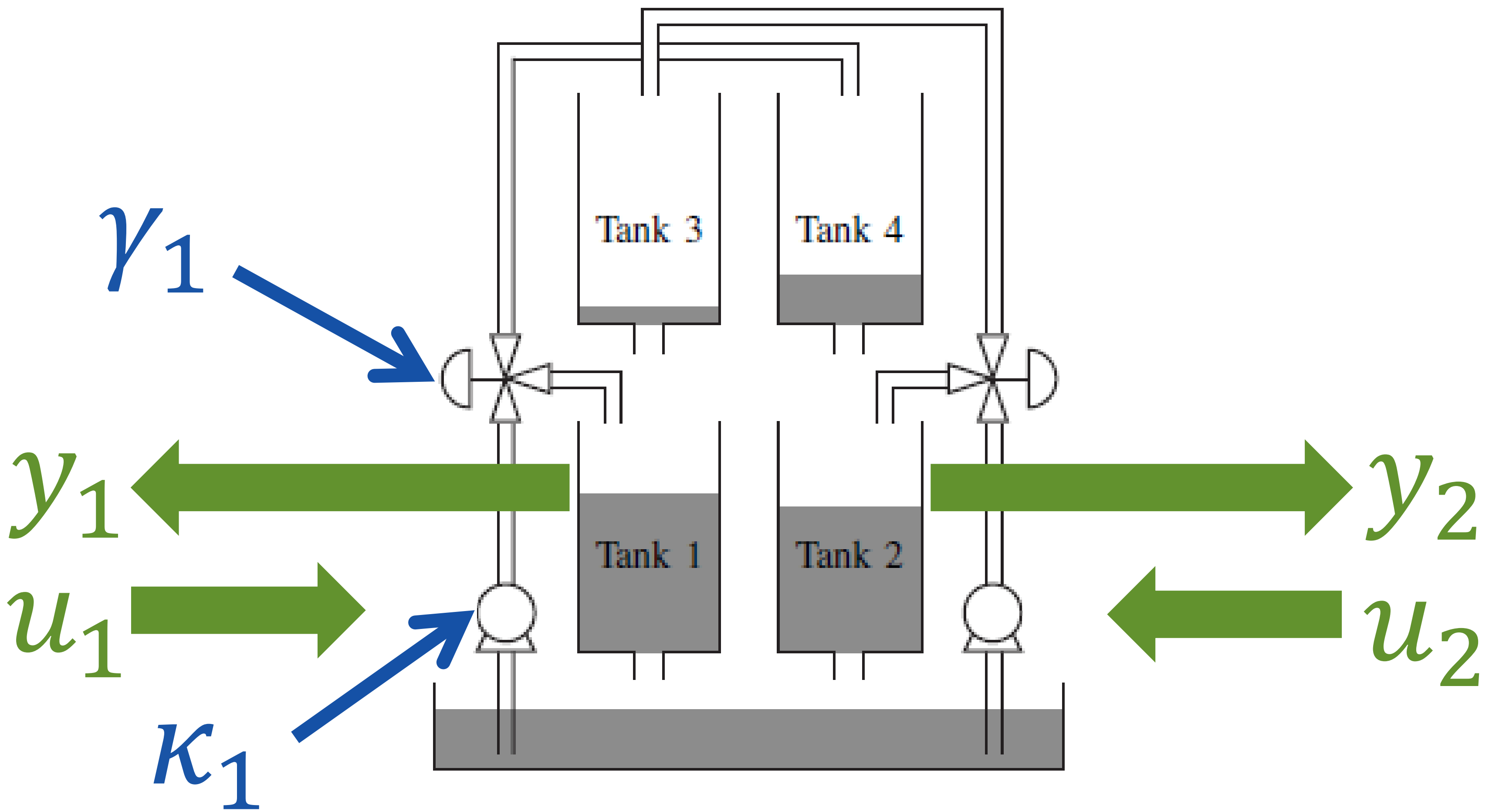
$$I_F \succcurlyeq \eta\gamma V''_{app}(\theta_0) \Leftrightarrow \mathcal{E}_{SI} \subseteq \mathcal{E}_{app}$$

Experiments

- MPC with integral action
- Reference tracking
  (Tank 1 & Tank 2)



Experiments (2 parameters)



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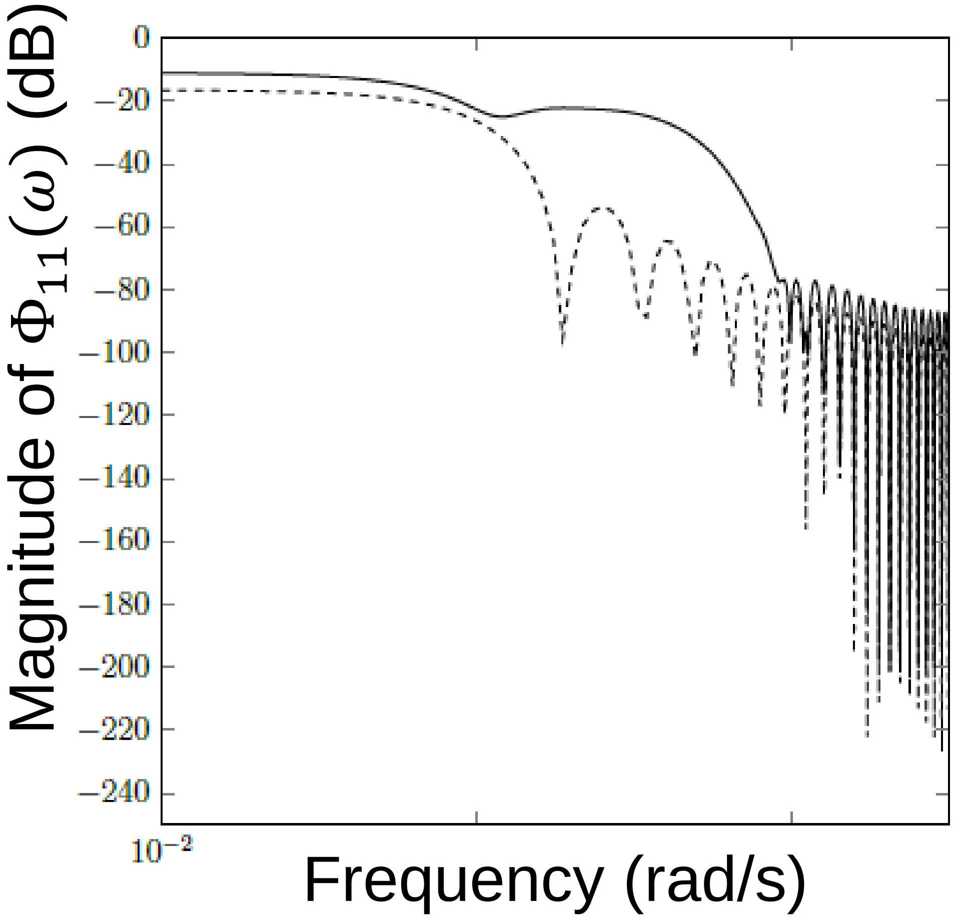

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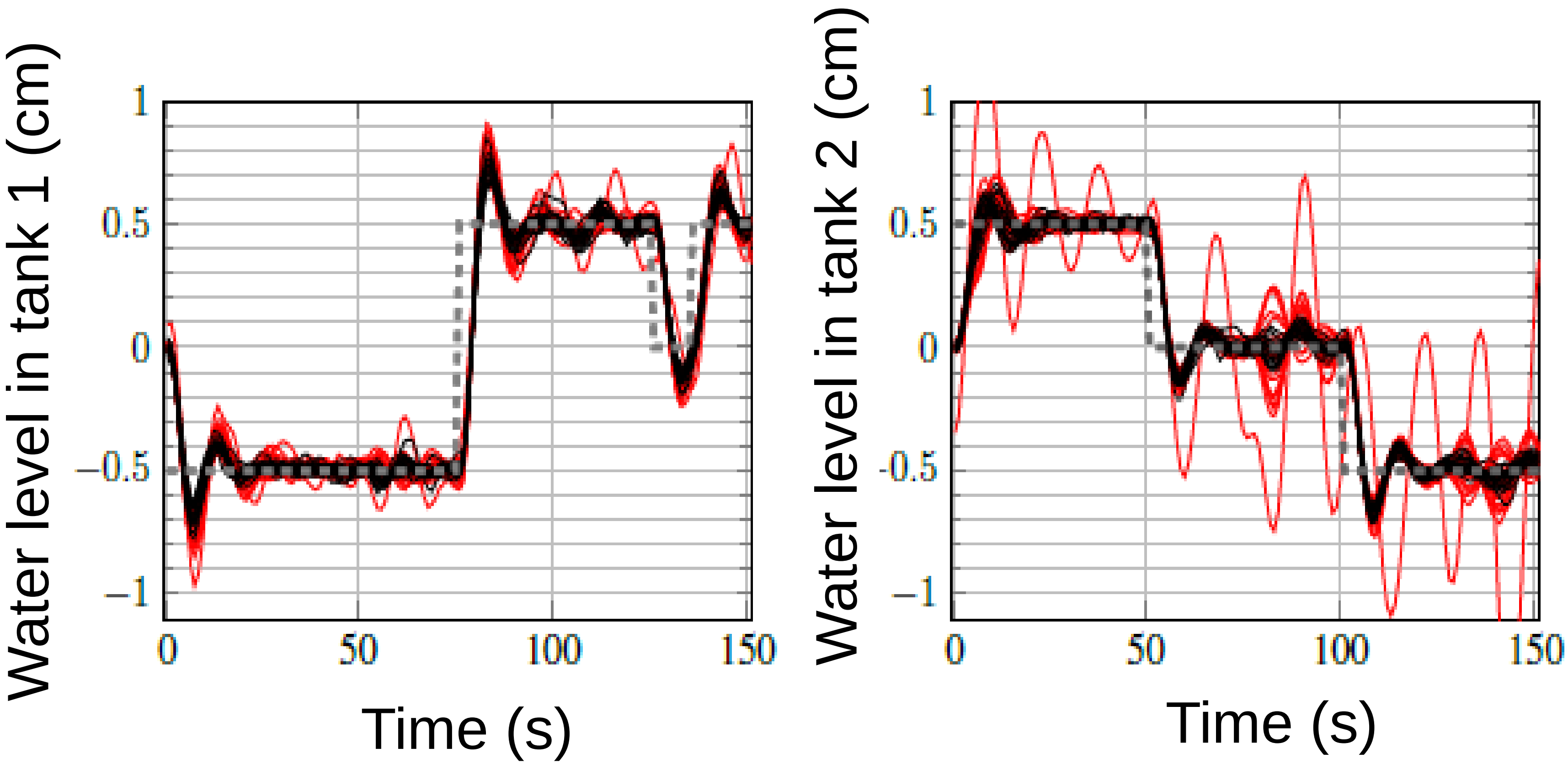
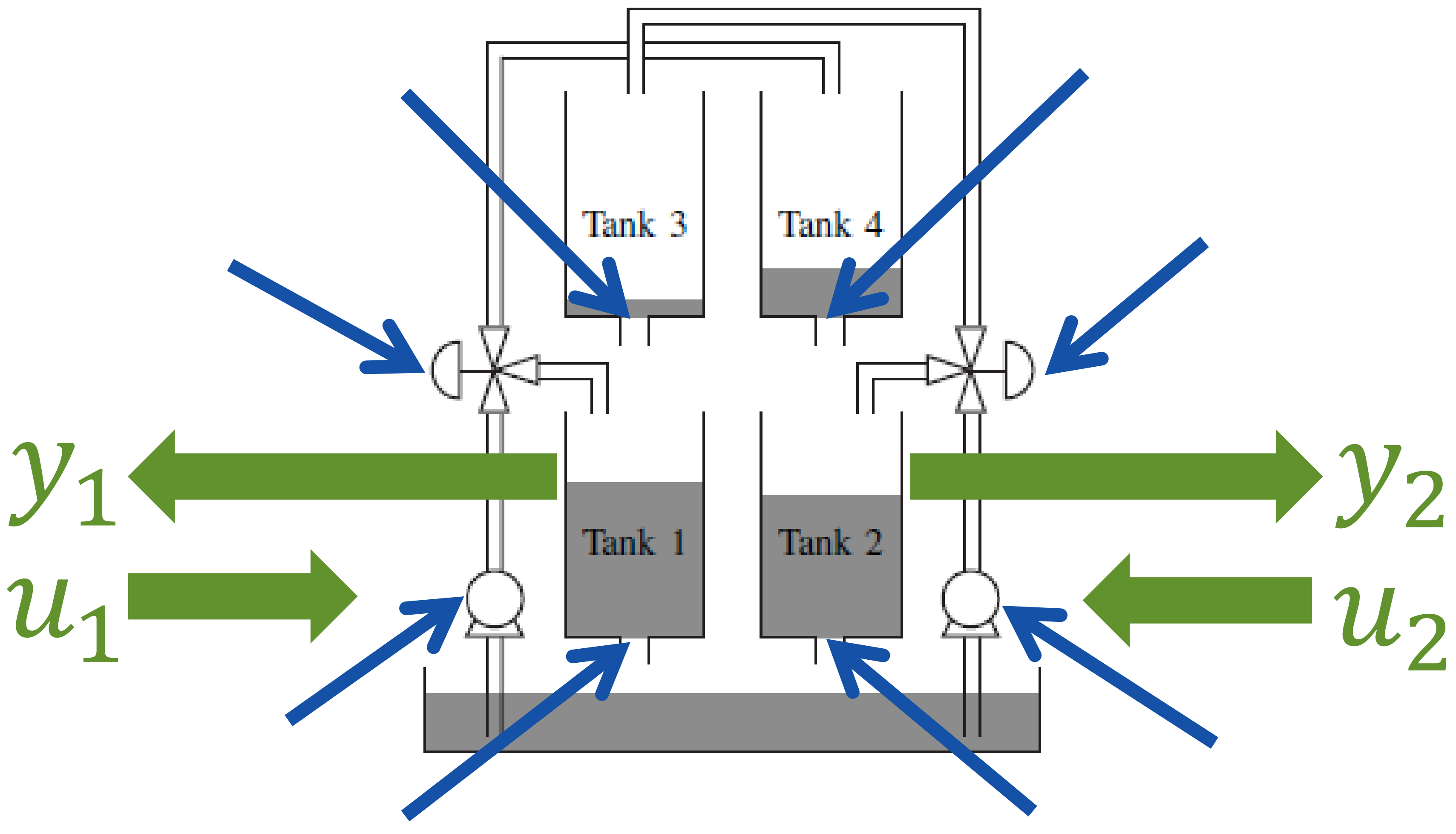


Experiments (8 parameters)



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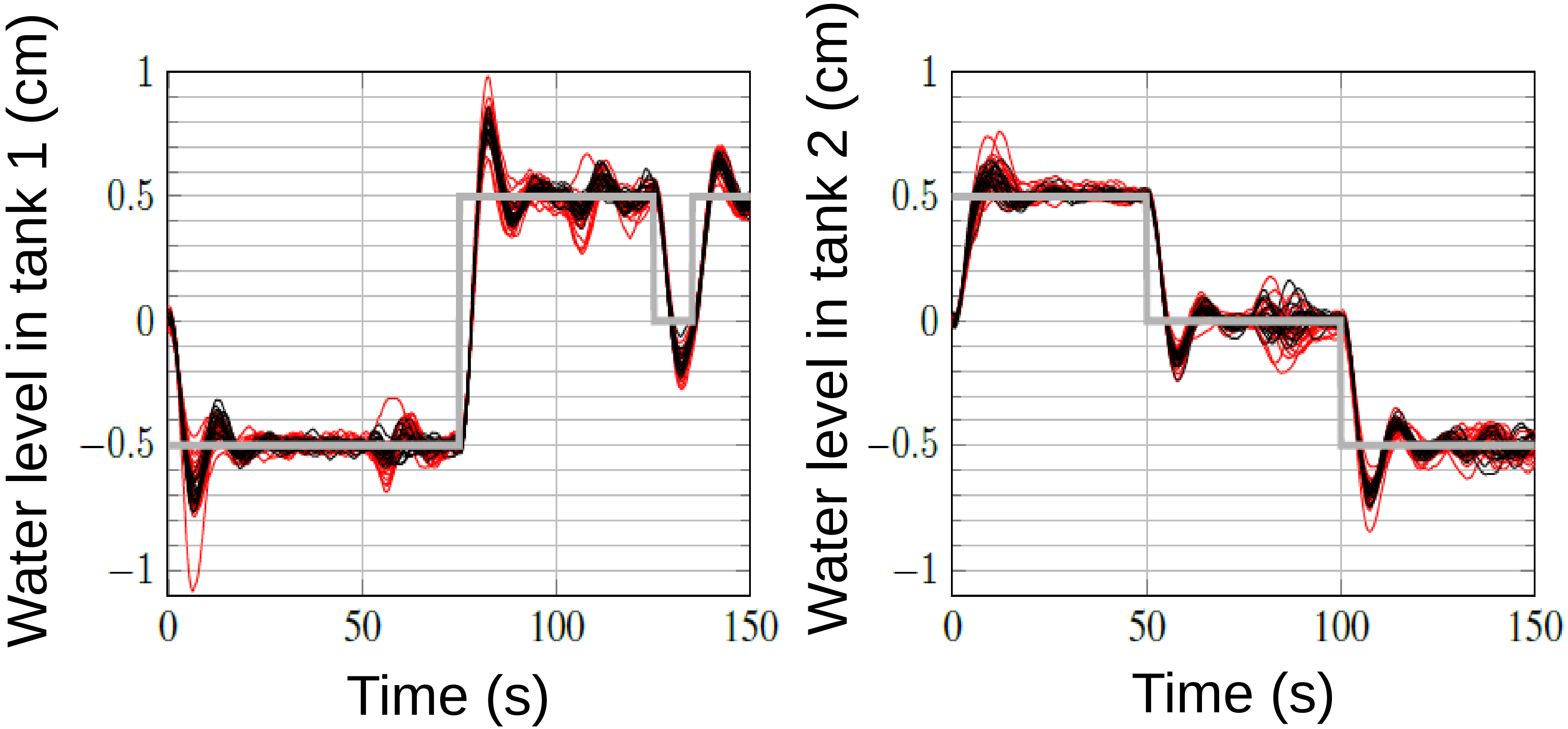

Thank you!